

On the generality of warrants

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ABSTRACT: A common problem in analysing and evaluating arguments is to determine what is implicitly assumed in drawing a conclusion c from one or more premisses P_1 through P_n . Some theorists of argument assert that the “logical minimum” is the argument’s associated material conditional, i.e. the “negajunction”: not (P_1 and .. and P_n and not c). Stephen Toulmin on the other hand maintains that every argument assumes a general warrant. I argue that Toulmin is correct: Every one-step piece of reasoning or argument from premisses P_1 through P_n to a conclusion c assumes the truth or acceptability of some generalization, possibly qualified, of its associated negajunction: not (P_1 and .. and P_n and not c).

KEYWORDS: warrant, generality, Stephen E. Toulmin, inference, inference claim, associated conditional, associated negajunction, inferential conditional

Introduction

In *The Uses of Argument* (1958: 98), Stephen Toulmin introduced to the theory of argumentation the concept of a warrant, conceives something produced by an arguer in response to the challenge, “How do you get there?” This challenge can arise after the arguer has adduced data in support of a claim, in response to the challenge, “What have you got to go on?” The question “How do you get there?” invites the author of the claim to explain how her or his data support it.

The answer, Toulmin says, will be a proposition of a different sort from the data. Rather than being an additional item of information, it will be a rule or inference-license. At its most candid and explicit, it will be expressed in such a form as, “Data such as D entitle one to draw conclusions, or

make claims, such as C.”

Toulmin’s warrants are not purely formal rules of inference. They are substantive rules. In his now famous hackneyed example—the argument “Harry was born in Bermuda, so Harry is a British subject”—the warrant is: That someone was born in Bermuda entitles one to conclude that that person is a British subject. Thus Toulmin’s warrants provide an alternative to the traditional, and still common, approach of assuming that arguments that are not formally valid have a missing premiss. On this more traditional approach, the task of the argument analyst is to supply this missing premiss, as it were to read the mind of the argument’s author, or perhaps to make explicit what the argument needs as a gap-filling premiss, whether or not the author actually had such a premiss in mind (Ennis, 1982).

I myself think that the search for such a missing premiss is generally a wild goose chase. Like the emperor’s new clothes, the missing premiss is not there. What is taken to be a missing premiss is in fact the articulation in statement form of a rule of inference that entitles one to draw the conclusion from the stated premiss or premisses.

Nevertheless, we can engage the proponents of the missing premiss approach without assuming that they are wrong and Toulmin is right. We can ask, “How do we determine what is the link between an argument’s stated premisses and its conclusion?” or more loosely “How does this follow?”

In addressing this question, I shall ignore Toulmin’s important concept of a qualifier, a word like “presumably” or “probably” that indicates “the degree of force which our data confer on our claim in virtue of our warrant” (Toulmin 2003: 93). Qualifiers and their associated defeaters are an important topic. But my focus is on identification of the warrant, or missing premiss if you like,

abstracted from its qualifier if any.

I shall also confine my attention to arguments in which the conclusion is an assertion, as opposed to a non-asserted speech act like a recommendation.

A distinctive feature of Toulmin's approach is that the link between data and claim is always general. Whatever the rule of inference on which an argument relies, it licenses not just the inference in that argument, but also analogous inferences in parallel arguments. It entitles us not only to infer from Harry's being born in Bermuda that Harry is a British subject, but also to infer from Marit's being born in Bermuda that Marit is a British subject, and so forth.

The missing premiss approach does not require any such generality. A missing premiss need only ensure the rejection of the situation that the premisses are true and the conclusion false. The simplest way to express such a rejection is just to assert it: It is not the case that the premisses of this argument are true and the conclusion is false.

The negation of the conjunction of the premisses and the negation of the conclusion is logically equivalent to the conditional statement that, if the premisses are true, then the conclusion is true, when this conditional statement is interpreted as a so-called "material" or "Philonian" or "truth-functional" conditional, i.e. as a conditional that is false where it has a true antecedent and a false consequent but otherwise true. In earlier work, I referred to this conditional as the argument's "associated conditional", and others have adopted this terminology. It is a bit misleading, however, to put the question whether an argument's gap-filling or inference-licensing assumption is general in terms of whether it is sufficient to supply the argument's associated conditional as a "missing premiss". For the truth-functional interpretation of ordinary-language singular indicative conditionals is highly implausible. When one asks if it is sufficient to take as the basis for the inference in the

Harry argument the conditional proposition “If Harry was born in Bermuda, then Harry is a British subject”, it is natural to respond affirmatively. That is because the word “if” in ordinary speech indicates an inferential relationship. As the ancient Stoics put it, “this connective [i.e. the connective “if”–DH] proclaims that the consequent follows from the antecedent” (Diogenes Laertius VII.71). But clearly the mere denial that one statement is true and another false does not mean that the second follows from the first. For example, it is not the case both that Mars is a planet and that Norway is not a Scandinavian country. But the geographical fact that Norway is a Scandinavian country does not follow from the astronomical fact that Mars is a planet. So it seems at least odd to say, “If Mars is a planet, then Norway is a Scandinavian country.” If one asked people in the street to say whether this statement is true or false, they would be perplexed as to how to answer. The natural reaction is to say that the two components have nothing to do with one another, so there is no way to tell whether one is true if the other is true. Anyone who knows that the consequent is true, i.e. that Norway is in fact a Scandinavian country, is likely to answer: “Well, Norway is a Scandinavian country, whether or not Mars is a planet. The fact that Mars is a planet has nothing to do with the matter.” The difference between our reaction to a conditional with such an irrelevant antecedent and our reaction to the corresponding negated conjunction tells against Grice’s view, in which he himself sees difficulties, that the inferential relationship implied by asserting a conditional statement is not a part of the meaning of “if”, but is non-conventionally implicated (Grice, 1989).

For clarity, then, I will phrase the question as to whether a merely particular statement is enough to indicate how an argument’s conclusion follows from its premiss(es) as a question whether it is sufficient to supply as a “missing premiss” or gap-filling assumption the negation of the conjunction of the argument’s premisses and the negation of its conclusion. For short, I will call the

negation of this conjunction the argument's "associated negajunction". The associated negajunction of the Harry argument is the proposition: it is not the case that Harry was born in Bermuda and Harry is not a British subject.

For further clarity, let me make clear that I am interpreting the words "not" and "and" in an argument's associated negajunction truth-functionally. That is, an "and" statement is true if and only if all the conjuncts joined by the word "and" are true, false if and only if at least one conjunct joined by the word "and" is false, and neither true nor false if and only if no conjunct joined by the word "and" is false but at least one of them is neither true nor false. Similarly, a negated proposition is true if and only if the proposition is false, false if and only if the proposition is true, and neither true nor false if and only if the proposition is neither true nor false.

Given this interpretation, one prove that the negajunction associated with a formally invalid argument, if added as an extra premiss, will produce a formally valid argument, and that it is the minimal addition that will do so.

To see that it will produce a formally valid argument, notice that the resulting argument will have the form: $P_1, \dots, P_n, \text{not } (P_1 \text{ and } \dots \text{ and } P_n \text{ and not } c), \text{ therefore } c$. Now suppose that this argument is formally invalid, i.e. that there is a re-interpretation of its extra-logical components in which the premisses of the expanded argument are true but the conclusion false. (We may suppose that the argument's premisses and conclusion have been put in some way into a canonical notation in which the grammatical structure corresponds to the logical structure and all commonalities of content among the extra-logical components have been made explicit.) On this interpretation, by hypothesis, the original premisses are all true (since they are premisses of the expanded argument as well) and the conclusion c is false (since it is the same as the conclusion of the original argument).

Hence the negation of the conclusion is true (since the negation of a false proposition is true). Hence the conjunction of the original premisses and the negation of the conclusion is true (since a conjunction whose conjuncts are all true is true). Hence the negation of this conjunction is false (since the negation of a true proposition is false). But on our hypothesis the negation of this conjunction is true, since it is a premiss of the expanded argument and was assumed to be true in the interpretation under consideration. Given this contradiction, our original supposition must be false. In other words there is no re-interpretation of the extra-logical components of the expanded argument in which the premisses of the expanded argument are true but the conclusion false. That is, the expanded argument is formally valid.

As a side remark, let me comment that I have spoken of a “re-interpretation” of the extra-logical components of the argument rather than an interpretation of them, since I am assuming that we are dealing with an actual argument in a natural language like English or Norwegian, suitably regimented so that its grammatical structure reflects its logical structure. One can of course go beyond that initial regimentation and symbolize the argument in a formal language with uninterpreted extra-logical constants, in which case one would be considering an interpretation of them rather than a re-interpretation.

To prove that an argument’s associated negajunction is the minimal added premiss that will make it formally valid, suppose that there is some logically weaker proposition whose addition to the argument will make it formally valid. Then, by definition of the concept of being logically weaker, this proposition is compatible with the truth of the negation of the associated negajunction. Hence there is a possible state of affairs in which the premisses of the original argument are true, the logically weaker added premiss is true, and the associated negajunction is false. But, if the associated

negajunction is false, then the conjunction of the original premisses and the negation of the conclusion is true (since a negated proposition is false if and only if the proposition being negated is true). But then the negation of the conclusion is true (since a conjunction is true only if all its conjuncts are true). But that means that the conclusion is false (since a negated proposition is true only if the proposition negated is false). Hence the expanded argument is not formally valid, since there is a logically possible state of affairs in which all its premisses are true but its conclusion is false. But this contradicts our original supposition, which must therefore be false. Hence there is no proposition logically weaker than an invalid argument's associated negajunction that can be added as an extra premiss so as to produce a formally valid argument.

So, from the missing premiss point of view, the associated negajunction is enough, and nothing less will do the job. What then justifies us in attributing to an argument or piece of reasoning that is not formally valid anything stronger than the associated negajunction?

First argument: need of the associated negajunction for support

One answer to this question is that the associated negajunction is in need of support. In general, it is not self-evident that we must deny that the premisses are true and the conclusion false.

Notice, however, that we would have logically watertight support for such an associated negajunction if we were in a position to deny a premiss of the argument or in a position to affirm its conclusion. In either case, we would be denying, by supposition with good justification, one of the conjuncts of the negated conjunction. And from such a denial there follows logically the denial of the conjunction, i.e. the affirmation of the argument's associated negajunction. But denying one of the conjuncts of the associated negajunction does not produce a satisfactory "missing premise". If

we add the denial of an explicit premiss, the expansion produces a logically inconsistent set of premisses, whose constituents cannot all be true, which means that the premisses cannot establish the truth of the conclusion—which in the typical case is the point of putting forward an argument or engaging in some reasoning. If we add the conclusion as a premiss, we simultaneously make the stated premiss redundant and the expanded argument uselessly circular. In general, whatever is attributed to the arguer or the argument as support for an argument’s associated negajunction must not *ipso facto* make the argument useless for the purpose for which it was advanced. Thus, for example, not only the denial of an explicit premiss but also any proposition or set of propositions that entail such a denial are unsatisfactory, and for the same reason: the resulting expanded set of premisses will be inconsistent. Similarly, not only the conclusion but also any proposition or set of propositions that entail the conclusion are unsatisfactory, because they simply displaces the stated premiss. A plausible candidate for an argument’s “missing premiss” must generate a flat contradiction from its combination with the stated premisses and the negation of the conclusion in some other way than logically implying all by itself either the conclusion or the denial of a premiss.

One family of candidates that obviously fill the bill is the set of generalizations of the associated negajunction. One can form such generalizations by taking one or more content expressions in the negajunction and generalizing over them. (By a content expression, I mean an expression that can be replaced without loss of grammaticality by an extra-logical constant, such as a proper name or a single-word predicate.) To take the Harry argument, we can get the following five basic generalizations of its associated negajunction:

content expression (in invisible quotation marks)	generalization of the associated negajunction
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Harry	For any person x , not both x was born in Bermuda and x is not a British subject. (= everyone born in Bermuda is a British subject)
Bermuda	For any place x , not both Harry was born in x and Harry is not a British subject.
born	For any relation R , not both Harry stood in relation R to Bermuda and Harry is not a British subject.
British	For any country x , not both Harry was born in Bermuda and Harry is not a subject of x .
subject	For any status S , not both Harry was born in Bermuda and Harry does not have status S in relation to Britain.

I call these generalizations “basic” because each is produced by generalizing on just one content expression in the associated negajunction with respect to the most obvious reference class. Each is subject to variation by varying the reference class; for example, a generalization over the content expression “Harry” with respect to male human beings would be: For any man x , not both x was born in Bermuda and x is not a British subject. In standard English, this generalization would be the proposition that every man born in Bermuda is a British subject. Further, one can generalize over more than one of these content expressions at a time. For example, generalizing over the content expression “Harry” with respect to human beings and over the content expression “Bermuda” with respect to British colonies produces the generalization: For any human being x and any British colony y , not both x was born in y and x is not a British subject. Or, in plain English: Everyone born in a British colony is a British subject. There is no limit in principle to the number of reference classes with respect to which we may generalize a single content expression; for example, we may generalize over the name “Harry” with respect to men, human beings, primates, mammals, vertebrates, animals, living organisms, physical objects, and so on. Hence the number of possible generalizations of an argument’s associated negajunction is quite large. It would be easy, for

example, to produce more than 100 generalizations of the negajunction associated with the Harry argument.

Any such generalization clearly implies the associated negajunction, provided one adds as assumptions that the reference classes with respect to which each content expression is generalized have as a member the individual, property or relation referred to by that content expression. For example, to derive the negajunction that not both Harry was born in Bermuda and Harry is not a British subject from the generalization that every person born in a British colony is a British subject, one needs the additional assumptions that Harry is a person and that Bermuda was a British colony when Harry was born.

(According to the theory of consequence that I have articulated elsewhere (Hitchcock, 1985; Hitchcock, 1998), an inference is non-defeasibly good only if some such covering generalization is non-trivially true, i.e. true for a reason other than the non-existence of an instance in which its antecedent is true or the non-existence of an instance in which its consequent is false. That is, there must be at least one instance parallel to the stated argument in which the conditions assumed in its premisses obtain and at least one instance parallel to the stated argument in which the condition inferred in its conclusion fails to obtain. This non-triviality condition is rooted in a rejection of the principles *ex falso quodlibet* and *e quolibet verum*; to take a medieval example, “Tom is in the corner” does not follow from “You are sitting and you are not sitting”. The non-triviality condition implies that there must be a true covering generalization that generalizes over at least one content expression that occurs in both a premiss and the conclusion.)

Is there any other way of providing satisfactory support to an argument’s associated negajunction than by supplying a covering generalization of the argument along with the required

additional assumptions about the reference classes with respect to which the generalization is made? It has been suggested (by Lilian Bermejo-Luque, in personal correspondence) that a reason for “if you promised, you have to do it” (construed as a material conditional, i.e. a negajunction) may be something like the very definition of “promising”. Similarly, a reason for “if the litmus paper turned red, then the liquid in which it was dipped is an acid” may be something like a chemical explanation. Definitions and chemical explanations, however, although they are not covering generalizations, entail covering generalizations, which may of course be modally qualified. So reasons of this kind imply that the author of the argument is committed to a covering generalization of its associated negajunction.

It seems that any support for an argument’s associated negajunction that would be a satisfactory missing premiss, in the sense that it neither makes the expanded premiss set inconsistent nor makes any original premiss redundant nor makes the expanded argument uselessly circular, either is or entails a covering generalization of that negajunction. If so, even on the missing premiss approach the author of a formally invalid argument is committed to the truth of some covering generalization of the argument’s associated negajunction.

Second argument: the practice of refutation by logical analogy

Thus far, I have been working within the missing premiss approach, trying not to beg the question in favour of the warrant approach. I would like to turn now to a more direct approach in terms of our everyday practice when we respond to other people’s arguments or reflect on the adequacy of our own reasoning.

Without any special tutoring in logic, people spontaneously use a method of challenging

inferences that the textbooks call “refutation by logical analogy”. They challenge the claim that a certain conclusion follows from the stated premisses by producing a logically parallel argument in which the premisses are true but the conclusion false. Children do this at quite a young age. Mother says: “Johnny, you can’t have any dessert, because you did not eat your peas.” Johnny replies: “But mummy, Mary didn’t eat her peas either, and you let her have dessert.” If mummy has any commitment to being reasonable in her interactions with her children, she will not reply: “I’m not talking about Mary, I’m talking about you.” She might say something like the following: “Well, Mary ate almost all her peas, but you haven’t touched yours”, thus implicitly revising the premiss of her original argument. Or she might say: “I shouldn’t have given Mary peas at all, because she is allergic to them. You don’t have any allergy to peas”, thus restricting the reference class of the covering generalization that children who do not eat their peas cannot get any dessert. Or she might simply concede the force of Johnny’s refutation and say, “OK, you’re right. You can have dessert.”

An adult might use a “you might as well say” construction to challenge someone’s inference. In the simplest cases, the parallel can be very direct, involving a substitution of just one content expression. Someone might respond to the Harry argument by saying, “You might as well say that Marit is a British subject, because she was born in Bermuda”, where it is known that, although indeed being born in Bermuda, Marit is not a British subject. The author of the Harry argument might then reply, “But neither of Marit’s parents was a British subject. They are both Norwegian citizens.” This reply can be construed either as limiting the scope of the covering generalization (“all people born in Bermuda with at least one parent who is a British subject are British subjects”) or as indicating that the inference is defeasible rather than definite.

In real life, refutation by logical analogy seems to operate at a higher semantic level than

simply replacing one content expression by another. Here is an example:

Magnets are not a power source! It appears that a magnet is energetic and powerful when something is attracted sharply toward it, but it always takes as much effort to pull the object back away. You might as well say that ski slopes are energy sources because you are pulled down them. (posted by Jeff Saxton on 23 August 2006 at <http://www.lastpodcast.net/2006/08/23/steorn/>; visited 27 December 2006)

The context of this posting is discussion of a claim by a company called “steorn” to have developed a technology that produces free, clean and constant energy. The published patent for the company’s device indicates that it is a “magnetic actuator”. Saxton’s “you might as well say” claim responds to a superficially attractive argument that magnets are a power source because some things are sharply attracted to them. This argument is not valid, he claims, because it takes as much effort to pull the attracted object back away from the magnet. Similarly, it takes as much effort to get the skier to the top of the ski slope as is gained when the skier goes down the slope. Thus we can set up the parallel arguments as follows:

Some objects are sharply attracted to magnets.	People are pulled down ski slopes.
Therefore, magnets are an energy source.	Therefore, ski slopes are an energy source.

The parallel argument has a true premiss but an obviously false conclusion: we cannot use ski slopes as an energy source because we have to use energy to get objects to the top of the ski slope first. So, if it is really a parallel argument, it shows that the conclusion of the original argument does not follow. (Note that it does *not* show that the conclusion of the original argument is false.) But is it a parallel argument? In some respects, it can be gotten from the original argument by straight

substitution, replacing “magnets” by ski slopes”. But in other respects there is a transference: “some objects” is replaced by “people” and “sharply attracted to” by “pulled down”. Such wholesale substitutions in general don’t produce a refutation by logical analogy. Consider for example an attempt to refute the Harry argument by objecting, “You might as well say that Marit is a British subject because Marit was raised in Norway.” Here the objector not only replaces “Harry” by “Marit” but also replaces “born” by “raised” and “Bermuda” by “Norway”. Intuitively, the objection is irrelevant: the status of a person raised in Norway has nothing to do with the status of a person born in Bermuda. In the Harry argument, we intuitively recognize that the required counter-example must be a person born in Bermuda who is not a British subject.

Why then is it apparently successful to use an argument from ski slopes pulling people down them to refute by analogy an argument from magnets sharply attracting objects toward them? The answer is that pulling down and attracting towards are both cases of initiating motion, which is the semantic component of magnetic attraction that leads some people to think that magnets are an energy source. Further, the difference between a metal object attracted by a magnet and a person pulled down a ski slope is not relevant; both are physical objects with a mass, and the initiation of the motion of some object looks like the transmission of energy. So, although the terms “pulling down” and “attracting” are not synonyms, they share a semantic component of initiating motion, which is the operative component for the purposes of the arguments in which they occur. Similarly, the terms “people” and “some objects” are not synonyms, but people are a kind of object, in the sense relevant to being an energy source. An energy source is something that can produce motion in a physical object, in the sense of an object with mass.

It would certainly not be an appropriate response to Saxton’s refutation by logical analogy

to say that the original argument concerned objects attracted by magnets, not people going down ski slopes.

Is an argument's inference claim its associated negajunction?

So far I have advanced two arguments in support of my thesis that every inference assumes the truth of some covering generalization. The first is that such a generalization provides the only satisfactory support for the minimal assumption needed for the conclusion to follow logically from the reasons from which it is inferred. The second is that the thesis is assumed by the common practice of refutation by logical analogy, whose relevance nobody challenges.

I now wish to turn to arguments given by those who hold that some arguments need only a particular conditional as a gap-filler.

In her recent paper "Toulmin's model and the question of relativism" (Bermejo-Luque, 2006), Lilian Bermejo-Luque proposes that "we should interpret the warrant as the corresponding material conditional" (p. 79), that is, as what I have been calling the argument's associated negajunction. She identifies the warrant with "the corresponding inference-claim of each argument" (p. 80), that is, with the claim that the conclusion follows from the premisses offered in its support. And she takes this claim, identified with the argument's associated negajunction, to be "the particular conditional that licenses the step from a particular reason to a particular conclusion" (p. 80).

As Bermejo-Luque herself acknowledges, this conception flies in the face of Toulmin's explicit statement in *The Uses of Argument* (1958, p. 98) that warrants are general. But I wish to focus, not on its hermeneutic adequacy as a description of Toulmin's model, but on its theoretical

adequacy as an account of what is involved in inferring a claim from a reason.

My basic response to Bermejo-Luque's proposal is that an argument's associated negajunction is not its inference-claim. To say that it is not the case that Harry was born in Bermuda and Harry is not a British subject is not to say that it follows from the fact that Harry was born in Bermuda that Harry is a British subject. The associated negajunction makes a much weaker claim than the claim implicit in an illative like "so" or "therefore" that introduces a conclusion. We can see that this is so by constructing an example where the associated negajunction is true but the inference-claim is false. Consider, for example, the argument, "8 is divisible by 2, so 8 is divisible by 4." This is clearly a bad argument, even though it has a true premiss and a true conclusion. You cannot legitimately infer from the fact that 8 is divisible by 2 that 8 is also divisible by 4, even though as a matter of fact it is true that 8 is divisible by 4. Thus the inference-claim in the argument "8 is divisible by 2, so 8 is divisible by 4" is false. But the associated negajunction "not both 8 is divisible by 2 and 8 is not divisible by 4" is true, because one of the two conjuncts in the negated conjunction, namely the conjunct "8 is not divisible by 4", is false, making the conjunction as a whole false and thus its negation true.

The claim that an argument's associated negajunction is its inference-claim gets its plausibility from expressing this negajunction as a conditional, "If the premisses are true, then the conclusion is true". Such a conditional clearly *is* the argument's inference-claim. But its truth-conditions are not those of the material or Philonian conditional. The conditional statement "if 8 is divisible by 2, then 8 is divisible by 4" is false, for exactly the same reasons as the argument "8 is divisible by 2, so 8 is divisible by 4" has a bad inference. The conclusion of the argument does not follow from its premiss; similarly, the consequent of the conditional does not follow from its

antecedent. This example shows as clearly as any that the truth-functional account of the truth-conditions for a particular indicative conditional in natural languages is incorrect. It may be convenient to translate natural-language “if .. then” particular sentences into a formal language with a horseshoe or arrow as the main connective, and to treat this connective as signifying a truth-function. But it is a convenient fiction.

My counter-example of course involves a case where the conclusion is known to be true independently of the argument put forward in its support. Bermejo-Luque tries to ward off this sort of counter-example by remarking that an argument’s associated material conditional (i.e. its associated negajunction)

is to be valued under the argumentative conditions in which it arises, namely, that the reason alleged in the argument (whose content is the antecedent of the conditional) is supposed to be true or highly acceptable, and also that we have not already independently determined the real value of the claim for which we argue (whose content is the consequent of the conditional). These conditions suffice to free us from the paradoxes of material implication when appraising warrants. (p. 79)

So we are to construe my supposed counter-example as an argument presented in a situation where it is supposed to be true that 8 is divisible by 2, but we have not already determined independently whether 8 is divisible by 4. To make such a situation plausible, let’s vary the example slightly to one where the arithmetical premiss is obviously true, the arithmetical conclusion not yet determined to be true, and the inference apparently incorrect. An example might be the argument: “20,472 is divisible by 3, so 20,472 is divisible by 9.” According to Bermejo-Luque, the fact that the premiss of this argument is supposed to be true (and can in fact easily be checked to be true) but that we have

not yet determined whether the conclusion is true (because we have not yet tried to divide 20,472 by 9) frees us from the paradoxes of material implication when we come to appraise the argument's associated material conditional, i.e. the associated negajunction "Not both 20,472 is divisible by 3 and 20,472 is not divisible by 9." But surely the easiest and most direct way to determine whether this negajunction is true, given that we know that the first of the two conjuncts in the negated conjunction is true, is to check whether the second one is also true. If the second conjunct is also true, then the whole conjunction is true, the associated negajunction is false and the conclusion does not follow. If the second conjunct is false, then the whole conjunction is false, the associated negajunction is true and the conclusion does follow. The fact that we have not already determined the value of our conclusion does not bar us, when it comes to appraising the associated negajunction, from making such a determination independently of the argument offered in its support. And, in any case where we have a way of making such an independent determination, and that determination yields the result that the conclusion is true, we will rightly conclude that the associated negajunction is true. Thus, on Bermejo-Luque's account, every argument whose conclusion can be independently determined to be true makes a true inference-claim: its conclusion does in fact follow from the premisses offered in its support.

This is a clearly unacceptable consequence, as we can readily see by constructing simple examples of arguments with an obviously true conclusion and an obviously irrelevant premiss. "Snow is white, so grass is green" is not a good argument, even though both its premiss and its associated negajunction are both known to be true. The conclusion that grass is green simply does not follow from the premiss that snow is white, which is obviously totally irrelevant to the conclusion. The word "so" implies, as part of its meaning and not as some pragmatic implicature of

its ordinary use, that the statement preceding it is relevant to the statement following, in the sense that it helps to establish the truth of the conclusion (Hitchcock, 1992).

My own position, as stated earlier in this paper, is that the inference-claim of an argument is the claim that some generalization of the argument's associated negajunction is non-trivially true (Hitchcock, 1985; Hitchcock, 1998). Any such generalization is equivalent in force to a Toulminian warrant, expressed as the claiming of an entitlement: "Data such as D entitle one to make claims such as C." (Toulmin, 1958, p. 98) One only has such an entitlement if claims such as C are true whenever we have corresponding data such as D. And conversely, if it is non-trivially true that there are no cases where we have data such as D but the corresponding claim C is not true, then data such as D do entitle us to make claims such as C.

In opposition to this sort of position, Bermejo-Luque argues (2006, p. 78) that general rules are not inference-claims, because they are not bridges between reasons and claims, for two independent reasons. First, several rules of inference are suitable for any given argument. Second, every general rule can have conditions of rebuttal applicable to the particular case stated by the argument. These two points are correct, and they tell against identification of an argument's inference-claim with some specific general rule. But they do not tell against identification of the inference-claim with the claim that some general rule or other licenses the transition from the reason to the claim. The question "does this claim follow from this reason?" is not identical with the question "is this particular covering generalization of the associated negajunction non-trivially true?" But it is identical with the question "Is some covering generalization of the associated negajunction non-trivially true?"

From this perspective, a specific warrant, such as "Information that someone was born in

Bermuda entitles one to claim that this person is a British subject”, acts as support for the more vague inference-claim, “Data such as the information that Harry was born in Bermuda entitle one to make claims such as the claim that Harry is a British subject.” Or, to put the same point in terms of generalizations of an argument’s associated negajunction, the non-trivial truth of a specific generalization of the associated negajunction, such as the generalization that everybody born in Bermuda is a British subject, provides support for the more general inference-claim that some generalization of the associated negajunction is non-trivially true.

Bermejo-Luque deploys an infinite regress argument against this attempt to construe a specific warrant as support for an argument’s inference-claim:

... if warrants justify the inferences, ... they should be reasons for the corresponding inference claims. But, if warrants “justify” our inferences in this sense, every argument contains another argument, namely the argument “warrant, so inference claim”. But then, we would need a new warrant to justify our inference from our warrant-reason-for-the-inference to our inference claim, and this warrant would be another reason with a new warrant to bridge the new gap, and so on. Thus, we would never be entitled to infer a claim from a reason if warrants, as bridges between reason and claim, should bridge the gap as justifications for our inferences. (p. 77)

On the account I am proposing, the inference claim is an existential generalization, and the warrant is an instance of this generalization, perhaps accompanied by information that the instance does in fact fall under the generalization (e.g. that Harry is a human being). Thus the account is subject to the opening step of Bermejo-Luque’s regress: every argument contains an argument “warrant, so inference claim”. The inference claim of such an argument is the claim of the non-trivial truth of

some generalization of the following associated negajunction: It is not the case that a warrant such as this is true and the corresponding inference claim is false. And the warrant supporting this inference claim is that every existential generalization with a true instance is true. This second-order warrant however is obviously true. It does not need justification. Further, the inference from it to the second-order inference claim is obviously valid. We could if we liked make explicit the third-order inference claim implicit in our argument, and the warrant supporting it. And there is no end to the possibility of further ascent. But such further ascent is not needed to show that the inference is justified. Thus Bermejo-Luque's infinite regress argument does not undermine my account of what is involved in inferring a conclusion from reasons offered in its support.

The extent of abstraction involved in each step of the regress may leave it unclear what exactly are the inference claim and warrant at each step. So I shall illustrate the sequence with our familiar example of the Harry argument:

First-order argument: Harry was born in Bermuda, so Harry is a British subject.

Inference claim: Some generalization of the claim that it is not the case that Harry was born in Bermuda and is not a British subject is non-trivially true.

Warrant: Every person born in Bermuda is a British subject, and Harry is a person.

Second-order argument: Every person born in Bermuda is a British subject, and Harry is a person.

So some generalization of the claim that it is not the case that Harry was born in Bermuda and is not a British subject is non-trivially true.

Second-order inference claim: Some generalization is non-trivially true of the claim that it is not the case that every person born in Bermuda is a British subject, and Harry is a person and no generalization of the claim that it is not the case that Harry was born in Bermuda and is not a British

subject is non-trivially true.

Second-order warrant: It is non-trivially true that every person born in Bermuda is a British subject.

Third-order argument: It is non-trivially true that every person born in Bermuda is a British subject.

So some generalization is non-trivially true of the claim that it is not the case that every person born in Bermuda is a British subject, and Harry is a person and no generalization of the claim that it is not the case that Harry was born in Bermuda and is not a British subject is non-trivially true.

Third-order inference claim: Some generalization is non-trivially true of the claim that it is not the case that it is non-trivially true that every person born in Bermuda is a British subject and that no generalization is non-trivially true of the claim that it is not the case that every person born in Bermuda is a British subject, and Harry is a person and no generalization of the claim that it is not the case that Harry was born in Bermuda and is not a British subject is non-trivially true.

Third-order warrant: It is non-trivially true that every existential generalization with a true instance is true.

The regress can continue, but it is not vicious. At an early stage, which may vary slightly from one person to another, both the truth of the warrant at that stage and the truth of the inference claim at the next stage are evident. We must not be so sceptical as to refuse to concede the truth of that which is perfectly obvious.

In personal correspondence, Bermejo-Luque has responded to this defence as follows:

... I think that the infinite regress is benign from an epistemological point of view... But it is not ok from a procedural point of view: the problem to conceive warrants as justifications is that warrants license the step from reason to claim. But if they do so by justifying this step, and justification is always a matter of having a sound argument, that is to say, an argument

that would need a warrant in this sense, then we would never be entitled to assert the claim because of the reason because we would never have a proper justification for our warrant, as this justification would require a justified warrant, and so on. In my view, the problem is to demand justification as a condition for justification. (Bermejo-Luquet to Hitchcock, e-mail message of 4 June 2005)

To this argument, I made the following reply:

This response strikes me as reasonable, but only if one thinks that every inferential move requires justification. I don't think that saying that warrants justify the inference implies that every inferential move requires justification. Obviously the latter proposition leads to a vicious infinite regress, unless one supposes that somehow the justification could be self-justification or circular. One could adopt the dialectical approach of Toulmin's book, taking it that one needs to supply a justification only if there is a challenge. One supplies grounds if one makes a claim and is asked, "What do you have to go on?" One supplies a warrant if one adduces grounds for one's claim and is then asked, "How do you get there?" I take it that, on your view, the applicability of the warrant could be challenged in something like the following way: "How does that get you there?", to which the answer would be that the warrant covers the inference claim. Theoretically, there is no limit to the series of such challenges. But the claim has been justified to a challenger if the challenger at some point stops making challenges and indicates that the series of responses to that point is sufficient. (Hitchcock to Bermejo-Luque, e-mail message of 4 June 2005, supplemented by subsequent clarification)

In the present paper, I have taken the alternative position that, at some early stage n of the regress,

the n th-order warrant and the $n+1$ th-order inference claim become self-justifying. This position is the equivalent in an individualist epistemology of the position in a dialectical epistemology that justification is only required in response to a challenge.

In the same correspondence, Bermejo-Luque has offered the following critique of my claim that the inference claim in an argument is a covering generalization of it:

... inference claims are not covering generalizations, in general. Take the case of ... arguments like “Socrates is hungry, so he didn't have breakfast this morning”. It is because of the features of the particular case stated by my reason (for example, that I'm talking of someone who usually has big breakfast, that it is only 10 o'clock in the morning, that he is not at home, or whatever) that I can come to determine the truth-value of the particular inference claim 'if Socrates is hungry, he didn't have breakfast this morning'. I do not need any cover[ing] generalization, although I can try to develop a suitable one, for example, for the sake of testing a hypothesis on Socrates' habits, on a person's need of food, or whatever. In my view, covering generalizations can justify the particular inference claim of an argument, but as justifications (and not as mere synonyms); they can also fail to do so. (Bermejo-Luque to Hitchcock, e-mail message of 4 June 2005)

The invented argument “Socrates is hungry, so he didn't have breakfast this morning” provides an occasion for me to elaborate on and clarify my general thesis in this paper. Such arguments are clearly occasional, in the sense that their meaning and value is closely tied to a particular occasion of utterance, i.e. a particular time and place, in the way that an argument in a written text meant for a temporally and spatially indefinite audience is not so closely tied to a particular occasion. To understand such an occasional argument, we need to know who uttered it, what particular individuals

are being referred to by its constituent proper names and definite descriptions, and what background knowledge about those particular individuals is being taken for granted as shared between the arguer and the argument's addressees. These "features of the particular case" stated by the reason are indeed necessary for evaluating the inference claim. But they are necessary in order to understand exactly what the inference claim is. The inference claim is not that "Socrates didn't have breakfast this morning" follows from "Socrates is hungry", but that "Socrates didn't have breakfast this morning" follows from "Socrates is hungry, and it is 10 a.m., and Socrates usually has breakfast before 10 a.m. but never so much before 10 a.m. that he would be hungry at 10 a.m." Once the content of the argument is spelled out in this way, it becomes plausible that the inference claim is that some covering generalization of the argument's associated negajunction is non-trivially true.

Thus the range of a covering generalization can be limited in accordance with the context of an argument. Further, the constant generalized over can be semantically implicit in the argument, e.g. in the tense of the verb. In the Socrates argument, there is an implicit reference to the time of the utterance, which let's say is 10 a.m. on June 4, 2005. Making this explicit, the associated conditional is: "If Socrates is hungry at 10 a.m. on June 4, 2005, then Socrates didn't have breakfast as of 10 a.m. on June 4, 2005." A generalization of this conditional is: "On any day when Socrates is hungry at 10 a.m., then Socrates didn't have breakfast as of 10 a.m." And the range of days can be limited to the days when Socrates is old enough to eat breakfast and is still alive.

Occasional arguments exhibit the kernel of truth in the missing premiss approach. Such arguments do need gap-filling supplementation by information about the topic of the argument (i.e. the individual person or thing referred to in both premisses and conclusion), information that is taken for granted as known by both arguer and addressees. But this supplementation usually does not

produce an argument that is formally valid. Rather, it produces an argument with an inference-claim that is at least arguably correct, in the sense that some justified warrant licenses the drawing of the conclusion from the supplemented premiss set.

Is an argument's inference claim a particular "inferential conditional"?

Another author who identifies the inference-claim of an argument with a particular, ungeneralized conditional is Bart Verheij. Verheij (2006, p. 186) takes an argument to express that its premisses collectively support its conclusion. And he takes this support relation to be expressed by the conditional sentence "if the premisses, then the conclusion". For example, the claim that the premiss "Harry was born in Bermuda" supports the conclusion "Harry is a British subject" is expressed by the sentence "if Harry was born in Bermuda, he is a British subject".

Unlike Bermejo-Luque, however, Verheij refuses to identify an argument's inference claim with the material conditional, i.e. with what I have been calling the associated negajunction. One reason for his refusal is that "a material conditional is truth-functional: its truth value is determined by the truth values of the conditional's antecedent and consequent", but "the conditional 'If D, then C' implied by an argument 'D. So C' should however intuitively reflect some relation between D and C that is not captured by the truth values of D and C alone." (p. 187)

Let us call Verheij's non-material, non-truth-functional particular conditional an "*inferential conditional*", since it expresses the condition that an argument's premisses support its conclusion, i.e. that it is legitimate to infer the conclusion from the premisses. Verheij does not give a complete account of the semantics of the inferential conditional. He tells us that it validates Modus ponens. In other words, an argument from an inferential conditional and its antecedent to the consequent of

the inferential conditional is formally valid. He tells us that one cannot derive an inferential conditional on the basis of logic alone (e.g. from a deduction of its consequent from its antecedent), but must always base its derivation on premises. But he says very little about what premises would entitle us to infer the truth of an inferential conditional. In fact, he claims that, if the logic of the inferential conditional were expressed in a natural deduction system, it would have the standard elimination rule for conditionals, namely *modus ponendo ponens*, but would have no introduction rule. The only derivation of an inferential conditional that he recognizes as legitimate is its derivation using *modus ponendo ponens* from a more complex inferential conditional of which the inferential conditional is the consequent. In particular, the inferential conditional associated with a particular argument follows from the argument's warrant, which is a conditional scheme expressed in ordinary language as a rule statement. We can illustrate this claim with our familiar Harry argument: The inferential conditional that Harry is a British subject if he was born in Bermuda follows from the statement that a person born in Bermuda is a British subject. The inferential conditional expressing that this warrant supports the original argument's associated inferential conditional is the statement: If a person born in Bermuda is a British subject, then Harry is a British subject if he was born in Bermuda. The consequent of this second-order inferential conditional is according to Verheij an instance of its antecedent.

Verheij needs to complete his account of the semantics of the inferential conditional. In particular, since a rule statement like "a person born in Bermuda is a British subject" sounds awfully like a generalized material conditional, i.e. a generalized negajunction such as "no person is born in Bermuda and is not a British subject", and the inferential conditional associated with an argument is supposed to be an instance of such a rule statement but not to be a mere material conditional,

Verheij needs to tell us how a warrant differs from a generalized negajunction. The answer, I suspect, will be an account like the one I have been developing in this paper.

Summary

Let me sum up.

I began by raising the question of the nature of the link between the stated premisses and the conclusion of an argument or piece of reasoning that is not formally valid. For the purpose of the exercise, I abstracted from the common occurrence of qualifiers like “probably” or “presumably” in our conclusions, and I confined my attention to arguments whose conclusion is an assertion. In particular, I raised the question whether the gap-filling assumption of such an argument needs to be general or can in some cases be the particular associated negajunction, the proposition that it is not the case that the stated premisses are true and the conclusion is not true.

I advanced two independent direct arguments for my thesis that all inferences are implicitly general.

First, I proved that the minimal assumption needed and sufficient to make an argument or piece of reasoning formally valid is its associated negajunction. But the associated negajunction is not self-certifying; it needs support. Such support can come from the denial of a premiss or the assertion of the conclusion, and in particular cases such a denial or assertion might be epistemically justified. But neither the denial of a premiss nor the assertion of the conclusion is a suitable candidate for an argument’s missing premiss, in the first case because it makes the expanded premiss set logically inconsistent and thus useless for proving the conclusion, and in the second case because it makes the original premisses redundant. In general, a satisfactory “missing premiss” must not only

entail the associated negajunction but also must be consistent with the original premisses and must make no original premiss redundant when it is added as an extra premiss to the original argument. I confessed an inability to conceive of any candidate that would meet these requirements other than some generalization of the associated negajunction, combined with assumptions that the reference classes with respect to which each content expression in the associated negajunction is generalized have as a member the individual, property or relation referred to by that content expression.

Second, I argued that the generally accepted practice of refutation by logical analogy assumes that the author of any argument is committed to parallel conclusions following from parallel premisses. Although an arguer can deflect an attempted refutation by logical analogy by narrowing the reference class so as to exclude the analogue or by claiming that the general feature licensing his inference relates to a different content expression than the one that is varied in the analogue argument, an arguer cannot simply reject the relevance of a parallel argument with true premisses and a false conclusion. In other words, the author of any argument is committed to the truth of some generalization of the argument's associated negajunction.

Having advanced these two direct arguments for my thesis that all inferences are general, I considered the opposing claim of Lilian Bermejo-Luque that the inference claim of an argument is its associated negajunction, by producing examples of arguments where the associated negajunction is true but the conclusion obviously does not follow from the premiss set. Three examples of such arguments were the following:

8 is divisible by 2, so 8 is divisible by 4.

20,472 is divisible by 3, so 20,472 is divisible by 9.

Snow is white, so grass is green.

On Bermejo-Luque's account, every argument with a conclusion that could independently be determined to be true would have a good inference, since its implicit inference claim would be true. This consequence is not avoided by her remark that appraisal of an argument's inference claim takes place in circumstances where the reason alleged in the argument is supposed to be true or highly acceptable and where we have not already independently determined the real value of the claim for which we argue.

I also responded to two arguments by Bermejo-Luque against construing an argument's inference-claim as a generalization, as well as to her infinite regress argument against treating warrants as justifications for an argument's inference claim. The inference claim of an argument, I said, is an existential claim, that some generalization of the argument's associated negajunction is non-trivially true. A warrant justifies this inference claim by providing an instance of the inference claim. The infinite regress involved in taking an argument's warrant to justify its inference claim is not vicious, since at an early stage no further regress is required for justification of the inference claim. Finally, I considered an argument by Bermejo-Luque that inference claims are not covering generalizations, since one needs information about a particular case to determine whether it follows, for example, from Socrates' hunger that he did not eat breakfast today. Such arguments, I said, are occasional, in the sense that their meaning and value is tied to a particular occasion of utterance. To understand such an occasional argument fully, we need to know who uttered it, what particular individuals are being referred to by its constituent proper names and definite descriptions, and what background knowledge about those particular individuals is being taken for granted as shared between the arguer and the argument's addressees. This fact about occasional arguments is the kernel of truth in the missing premiss approach. But, once the required particular facts have been supplied,

the inference claim in the now fully explicit argument is still the claim that some covering generalization is non-trivially true.

Finally, I turned to the contention of Bart Verheij that the inference claim of an argument is its ungeneralized associated conditional, construed not as a material conditional or negajunction but as what I called an inferential conditional. I argued that Verheij's account of the semantics of an argument's associated inferential conditional is incomplete, and that his contention that it follows from the argument's warrant, of which it is an instance, makes it imperative for him to clarify how it differs from the material conditional, since the warrant can easily be construed as a generalization of the material conditional. I speculated that an adequate semantics for Verheij's inferential conditional would produce an account that coincides with my own in the present paper.

To sum up the summing up: Every inference is general.

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