Chapter 1: The problem of time in classical philosophy

In this chapter we will be reviewing some of the long-standing issues about time and it passage that are relevant to the treatment of time in modern physics. This will take us into some territory that will be unfamiliar to most physicists, and indeed some of the historical content may be unfamiliar to many modern philosophers of science too. Part of the point of beginning with such a historical chapter is to provide valuable context for the discussion to follow. But in following these rich veins of traditional thought about time we will also find many precedents for contemporary views, including some deep-lying confusions about time and its passing that have persisted into contemporary thought. It is therefore important to try to resolve these lingering confusions here, before moving on in later chapters to a treatment of the more recondite issues that are specific to modern physics.

1. Difficulties concerning the reality of time and passage

Reflection about the nature of time seems to date from the early fifth century BCE, when thinkers in Asia Minor began the process of emancipating cosmology from its anthropomorphic trappings. This can be seen in the views of such men as Pherecydes of Syros and Anaximander of Miletos, when they construed time (*Chronos*) as an uncreated first principle, or described time as effecting a kind of equilibrium between opposing tendencies.[1] Within a mere two centuries of these beginnings, though, the Greeks had already achieved a high level of sophistication in thought about time. For by the time of Aristotle (384-322 BCE), students in the burgeoning schools were occupying themselves with such abstract problems as the reality of time, the status of the 'now', and the continuity of time.

Thus when Aristotle began his discussion of time with a summary of the main difficulties that needed addressing, the latter were three of the main difficulties about time that he recorded. First he reported considerations that would make one suspect "either that it does not exist at all, or at least that its existence is tenuous and faint" (*Physics* iv, 217 b32). Then he proceeded to questions about the status of the now: is the now always the same thing, or is it always different? Either supposition seems to lead to paradox. If it is always different, a now which has ceased to exist

[1] Pherecydes, who according to Aristotle wrote in a semi-mythical vein, claimed that "Zas [Zeus] and Time always existed", and Anaximander is reported to have said that "the things from which existing things come into being are also the things into which they are destroyed, … *for they give justice and reparation to one another for their injustice in accordance with the arrangement of time*". (Barnes 1987, 58, 75)

must have ceased to exist at some earlier now, but two different nows cannot be simultaneous. If it is always the same, then earlier and later events will be occurring at the same now, and "nothing would be either earlier or later than anything else". (*Physics* iv, 218 a8-29). Third, there are puzzles raised by Aristotle's claim that the 'now' is an indivisible bound separating past from future, and at the same time binding them into a continuous whole. This led many authors—including Diodorus Cronus shortly after Aristotle, Islamic theologians in the eighth century, and several Cartesians in the seventeenth—to claim that time consists of indivisibles or time atoms, so that it is not in fact continuous.[2]

Intriguingly, these roughly correspond to three of the main difficulties concerning time that are of interest to modern physicists and philosophers. Thus Julian Barbour and Carlo Rovelli have claimed that the way forward for reconciling the two great theories of modern physics, quantum theory and relativity, is to acknowledge the *unreality of time* that is signalled by its absence in the fundamental equations of modern physics. Regarding the second difficulty mentioned by Aristotle, the great majority of modern physicists and philosophers have concurred that there is *no room for the 'now'* in the modern physical worldview. Like 'here', 'now' is not something that features in the equations of physics. The events at the points of spacetime are all equally real, so the classical idea of reality coming into being by one set of events occurring 'now', to be succeeded by another set of events becoming at a later 'now', seems not to feature in such theories.[3] As we shall see, these difficulties are further compounded by the relativity of simultaneity in Einstein's special theory of relativity where, in Eddington's memorable phrase, there are no "world-wide instants" (Eddington 1929, 47). Philosophers, meanwhile, have their own reasons for being sceptical about the idea of a moving now. Movement, they object, surely presupposes time, so that the movement of the 'now' from earlier to later seems to presuppose another dimension of time, and to begin an impossible infinite regress of times.[4] Concerning the status of the now as constituting the continuity of time, again there is skepticism among both physicists and philosophers. It has been suggested by

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[2] See Sorabji 1983 for an engaging and scholarly account of theories of time, creation and the continuum in antiquity and the early middle ages. His book is especially valuable for its accounts of Aristotle's views, his treatment of time atomism in antiquity, and his comparisons of these theories with modern views.

[3] "The universe", writes Jack Smart, "is a four-dimensional space-time manifold. Present, past and future are all equally real." (Smart, 1968, 255); similarly the physicist Paul Davies: "all events—past, present and future—are equally real" (Davies 1995, 260).

[4] We should note that not everyone has regarded such an infinite regress of times as impossible. See footnote 12 below.

proponents of each of the two main approaches to a theory of quantum gravity, String Theory and Loop Quantum Gravity, that a theory of gravity consistent with quantum theory will require us to reject the continuity of time. Modern philosophers, meanwhile, have generally been skeptical of attempts to locate becoming in an instantaneous tendency to change state. This idea, found in classical thinkers such as Leibniz and revived by the neo-Kantian philosopher Hermann Cohen, has met with scathing rebuttal by those familiar with the mathematical theory of the continuum developed by Weierstraß, Dedekind and Cantor. Infinitesimals, Bertrand Russell insisted in the early 1900s, were banished from mathematics by that theory. Motion consists in being in one state at one time, and a later state at a later time, with no need to presume any such thing as a passage from one to the other. Modern philosophers of science have mostly agreed: the understanding of time as involving passage from one state to another is generally rejected in favour of such an austere "static" view.

In this book I am taking a different line. I claim that when becoming or the passage of time is correctly understood, there is nothing in modern physics (or mathematics, for that matter) to militate against it. To this end, I will now begin with a review of objections to the reality of time and passage that arise in classical physics and philosophy, postponing treatment of the further issues raised by relativistic and quantum physics to later chapters. It is fitting to begin with Aristotle, since, as we saw above, several of the objections just outlined have analogues in classical antiquity and can be resolved in a purely classical context; and, on the other hand, there are distinctions made by authors in the Aristotelian tradition, such as that between time and duration, which, although neglected in contemporary debates, have continuing relevance to modern issues, such as the distinction between co-ordinate time and proper time of relativity theory, which we will come to in chapter 3.

So let us take up in order the three problems mentioned by Aristotle: the reality of time, the status of the 'now', and the continuity of time.

2. Aristotle and the classic arguments for time's unreality

The argument for time's unreality recorded by Aristotle runs as follows:

> Some of it is past and no longer exists, and some is in the future and does not yet exist; these constitute both infinite time and the time that is with us at any moment; but it would appear to be

> impossible for anything which consists of nonbeings to participate in being itself. (*Physics* 217b 34-218a 3)

A more succinct version of this argument was later given by the influential medieval English philosopher William of Ockham (c. 1288-1348):

> that which is composed of non-entities is not a positive entity; but time is composed of non-entities, because it is composed of the past which does not exist now, although it did exist, and of the future, which does not yet exist; therefore time does not exist. (Ockham 1984, 496.)

But what of the present, it may be objected? Surely that exists now! Here another set of objections was already common property in ancient times. If the present has any extent, then part of it will be past and part future, so that the same argument will apply to those parts. Only the instant dividing the past and future, the instantaneous present, can be said to exist now. But this is not properly a part of time, since it is only a boundary between past and future, binding them together into a continuous duration. Thus the only parts of time are the past and the future, and neither exist.[5]

This argument for time's unreality based on the non-existence of its parts has had a long history. It was repeated by most commentators on time in the seventeen centuries between Aristotle and Ockham, including the African bishop Augustine (354-430) and the great Arab philosopher Averroës (Ibn Rushd, 1126-1198). "Time is composed of past and future," declared Averroës, "but the past has already stopped being and the future does not yet exist." (Duhem 1985, 301) And as late as the seventeenth century Gottfried Leibniz (1646-1716), Newton's main rival in natural philosophy, did not hesitate to employ a version of it against the absolute time which Newton supposed to exist independently of things. As he wrote to Newton's defender Samuel Clarke,

> Everything which exists of time and duration, being successive, perishes continually. And how can a thing exist eternally if, to speak precisely, it never exists at all? For how can a thing exist if no part of it ever exists? Nothing of time ever exists except instants, and an instant is not even a part of time. Anyone who considers these observations will easily comprehend that time can only be an ideal thing. (To Clarke, V, §49: GP VII, 402)

As remarked above, modern physicists have arrived at some conclusions that sound remarkably similar. Julian Barbour and Carlo Rovelli have urged physicists to realize that time is not part of

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[5] Cf. Aristotle: "time has parts, and some of them have existed, while others will exist, but none of them currently exist. The now is not a part of time... The now is a limit." (*Physics* iv, 218 a5, a23)

fundamental physical reality, and in chapter 6 we will be considering these authors' more technical arguments for this based on the Wheeler-DeWitt equation, regarded as fundamental in quantum gravity. But since these physicists, especially Barbour, also appeal to classical precedents for their views, let us look at their views in relation to these precedents first.

Taking inspiration from Leibniz's championing of time as a relation among things, as well as, perhaps his claim in the passage above that "nothing of time ever exists except instants", Barbour writes:

> We have been exploring Leibniz's idea that only things exist and that the supposed framework of space and time is a derived concept, a construction from things. If it is to succeed, the only possible candidates for the fundamental 'things' from which the framework is to be constructed are configurations of the universe: Nows or 'instants of time'. They can exist in their own right: we do not have to presuppose a framework in which they are embedded. In this view, the true arena of the world is timeless and frameless—it is the collection of all possible nows. (Barbour 1999, 177)

This last claim about the "true arena of the world" being timeless is confessedly indebted to Aristotle's teacher, Plato (who in turn was indebted to Parmenides). For in his *Timaeus* Plato had advocated the Parmenidean view that while Being is real, Becoming is but an illusion. Recounting this in a recent popular exposition of his views, Barbour states the ambition of going one step further, aiming to show how the illusion of time is produced. On his view, the world consists exclusively of what he calls "time capsules" or "instants", each of these being a concrete particular containing, in an implicit way, traces of its entire past history (Barbour dubs this conception of reality "Platonia".) These instants are "worlds unto themselves", "no thread of time joins them up." (Barbour 1999, 45ff). All we ever experience, Barbour proposes, we experience in an instant. The illusion of motion and of passage is produced by the circumstance that at any instant, "my brain contains several 'snapshots' at once", and "through the way in which it presents data to consciousness, it somehow 'plays the movie' for me". (267) Of course, there's a lot packed into that "somehow", since the very notions of presenting data and playing movies in the brain are both processes requiring not only time, but a thread linking the instants together. This is a notorious sticking point for such static views of reality: how to account for the illusion of passage or temporal becoming without presupposing the becoming of the illusion.[6]

[6] See Abner Shimony (1988) for criticism of this aspect of Barbour's position.

But we needn't dwell here on the shortcomings of Platonism as a philosophy of time. It will be enough to draw attention to a fundamental ambiguity that vitiates Barbour's position. This can be done by comparing it to the views of classical authors who argued for time's unreality along the lines we were considering above. For they seem to have taken the unreality of time in a sense that is a good deal less radical than Barbour's "timeless" interpretation. Neither Aristotle nor Averroës, for example, denied the reality of motion, nor did they believe that succession was illusory. Not even Ockham, perhaps the most relentless critic of the view that time is a kind of thing existing independently of things, denied that motion occurs or that the states of persisting things exist successively. So, given the alacrity with which many modern physicists have jumped from the premise that time does not exist independently of things to the conclusion that it must be eliminated from fundamental physics, this is an ambiguity that we should certainly consider further.

According to Aristotle's famous definition, adopted by Averroës and Ockham, time is the number or measure of motion. As such, it requires a soul to do the counting or measuring; but neither the motions it counts, such as the revolutions of the sphere containing the fixed stars, nor the succession of these revolutions, depend on being measured. On this understanding time is a kind of concomitant of motion, an aspect of the changes and motions we see about us. As Aristotle wrote, "time cannot exist without change" (*Physics* 219a 1-2). It has no independent reality, but nonetheless presupposes changes, motions and successions that do exist independently of us.

In much of his writing, Barbour speaks in exactly this vein. He speaks of the inspiration he received from the writings of the great Austrian physicist Ernst Mach, whom he quotes to this effect: "'It is utterly beyond our power,' he said, 'to *measure* the changes of things by *time*. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things"' (Barbour 1999, 67; Mach 1919, 224). This fits well with Barbour's masterly analysis of classical time in his (1989), where he describes Newton as correctly perceiving that beneath the various relative times measured by the motions of the heavenly bodies, there must be an equable time by means of which they can be correlated. That is, although there is not necessarily any body performing the equable motion corresponding to Newton's absolute time, it nevertheless has a measure, and "is, for all practical purposes, identical to the astronomers' ephemeris time" (1989, 633).[7] Time in this

[7] Barbour insightfully observes that it was Ptolemy who "prepared astronomy for the day when it had to be recognized that there is no motion at all that realizes the concrete the 'uniform flow of time'. … Nearly two millennia

sense is a construction with a sound empirical basis.[8] It is not a real thing, since it does not exist independently of things. It is the non-reality of time in this sense, it seems, that Barbour and Rovelli should be committed to. Once there is motion and change, on such a Machian view, it is not necessary also to posit time as a fundamental feature of reality. Insofar as it is something distinct from change, it is what we abstract from the changes around us.

So the theory of time maintained by Aristotle, Averroës and Ockham, and indeed Barbour in his Machian mode, is profoundly different from the radical view implicit in Barbour's Platonia. It is one thing for time to be derivative, a concomitant of change, as it is on the Machian view, and quite another for reality to be fundamentally changeless and timeless, as it is on the view implied by Barbour and Rovelli when they talk of the "elimination of time". Where Ockham and Leibniz deny the reality of time as an entity distinct from persisting things, and construe time as depending on the supposed successions of states and the motions of such things, Barbour claims that all such successions and motions, as well as any times we might abstract from them, are simply illusions.

It's worth dwelling on this point a little longer. For one might suspect that Barbour's Platonist view seems more consistent in the following sense. How, if time is a mere abstraction, could there be things persisting through time? Conversely, if there really are things persisting through time with states succeeding one another, then surely time cannot be a mere abstraction, but must be something real? Here it is important to recognize a crucial distinction made by authors in the Aristotelian tradition that allows them to evade this dilemma, the distinction between *time,* on the one hand, and *duration* on the other. Time for them was an abstraction made from motions, in fact, a measure. But substances themselves would still endure, and still go through successions of qualities or states in the absence of any such measure being applied. A day, for example, is a measure of time derived from the rotation of the Earth on its axis (or of the Sun around the Earth, on the older geocentric view). Duration, on the other hand, is something concrete, an attribute of an enduring thing.[9] Thus Ockham subscribed to an austere universe whose only denizens are

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after he died, in the nineteenth and present centuries, astronomers did in fact construct an abstract time in this sort of fashion. This time, called *ephemeris time*, is the time according to which the tables and positions of the planets and moon are calculated." (Barbour 1989, 181) A similar point about Newton's absolute time being the time constructed by astronomers was made independently in (Arthur 1995).

[8] Cf. (Barbour 1989, 181-2): "ephemeris time is abstract in the sense that it is not realized by any one particular motion, but it is concrete in the sense that it must be determined empirically from actually observed motions."

[9] In this tradition, an enduring thing is one that has a duration, and for something to perdure just for it to persist through a (non-zero) duration. Contemporary philosophers, following David Lewis, distinguish between *enduring* objects—three-dimensional existents that remain the same through time—and *perduring* ones, which have a four-

perduring things. He sought to show that a statement ostensibly referring to time could be reduced to statements that made no appeal to time as an independently existing entity. Accordingly, a statement that two processes last for the same time could be parsed in terms of a coincidence between the endpoints of their durations, and a measure of the length of this time would involve comparison with the duration of a further motion taken as standard—such as that of the Sun's apparent rotation.

The same distinction between the duration of substances and time as an abstract measure occurs in Descartes' writings: "In order to measure the duration of things, we compare their duration with the duration of the greatest and most regular motions which give rise to years and days, and we call this duration 'time'".[10] Leibniz also upheld this distinction. Opposing Newton's identification of time with duration, he wrote to Clarke: "Everything has its own extension, its own duration, but does not have its own time and does not keep its own space." (5th Letter, §46). Duration and extension were for him attributes of enduring and extended things. Where for Newton absolute time, "without reference to anything external, flows uniformly and by another name is called duration" (Newton 1999, 408), for Leibniz time is "the order of successive things", a construction from relations among substances that are assumed to have successions of states.

Because Newton's views about space and time prevailed over those of his rivals, this distinction between time and duration has become largely forgotten. But as we will see in later chapters, there is a sense in which it prefigures the later distinction introduced by Minkowski between co-ordinate time and proper time. Co-ordinate time involves the choice of a reference frame, including an origin and a measure of temporal interval, and allows us to determine the time elapsed (in that frame) between one event and another. Proper time, on the other hand, is specific to a particular process and its path: the proper time along the path measures the duration of the process, and is independent of the frame of reference. Granted, there are profound differences between these modern notions and the classical ones: both co-ordinate time and proper time involve measures (presuppose a metric), whereas classical duration did not; and according to relativity theory, the co-ordinate time elapsed between two events varies with reference frame in a way that no classical

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dimensional existence. A substance-at-a-time would just be an instantaneous time slice of such a perduring object. I will make no use of this distinction here, and use 'perdure' in its classical sense of to persist through a duration.

[10] Descartes, *Principles*, I §57. In fact Descartes went so far as claim that the duration of a thing and its continued existence as a substance are just two ways of describing the same thing: "since a substance cannot cease to endure without also ceasing to be, the distinction between a substance and its duration is merely one of reason" (*Principles*, I §62).

author dreamt of. Nevertheless, the fact that there is an invariant proper time specific to any given process means that in a relativistic world we can talk about real successions and durations even in the absence of a unique time co-ordinate for the whole of spacetime.

Before we leave the topic of the reality of time, though, there is another feature of the classical argument for time's unreality from the unreality of its parts that it is worth our attention. For one may wonder how Aristotle and Averroës are entitled to hold that time is the measure of motion, given the argument they gave against time's reality. If all that exists is present, then past and future motions also do not exist. And if one also denies, as did Aristotle, that there can be motion in an instant, then motion and change seem just as unreal as time. Past motions do not exist, future motions do not exist, and there is no motion in the present instant.

As it turns out, we can readily resolve this conundrum. But an understanding of the resolution is pivotal for discussions of time's reality, and we will be returning to it many times in what follows. It turns on recognizing that there is an ambiguity in the word "exists". When we say the past and future do not exist, we mean they do not *exist now*. But when we refer to the existence of things in time—for instance, whether dinosaurs existed when the first humans evolved—we are talking about whether they *existed at that time*. The first landing of people on the moon does not exist in the sense of occurring now, but when cranks contend that it never really happened, it is its occurrence on July 20, 1969 that is in contention. Neither past nor future motions exist now, but if they really occurred or will occur, they did or will exist at the times of their occurrence. Past motions are those that existed during past times, putative future motions are those that will exist at future times, even though neither exist now. Thus so long as we are clear about the sense in which we are using the word 'exists', there is no confusion.

The classical argument for time's unreality from the unreality of its parts itself trades on conflating these two different senses of the word exist. The crucial premise in that argument is that "all that exists is present", i.e. that all and only things that are present exist. This is a very pervasive tenet, known in the trade as *presentism*. It has been subscribed to by any number of major thinkers, from Aristotle through Ockham, Hobbes and Leibniz to McTaggart and many contemporary philosophers.[11] Now it may be thought that any argument that persuades the likes of Aristotle, Ockham and Leibniz must have considerable force. But if we take the word 'exist' in the sense of

[11] See for example Sider 1999, Hinchliff 2000 and Markosian 2003 for modern defences of presentism, and Dorato 2006 and Savitt 2006 for criticisms.

'exist now', the presentist tenet reduces to the truism: "all and only things that are present exist now", i.e. "all and only things that exist now exist now". If, on the other hand, we interpret 'exist' as 'exist at a certain time', then past events (assuming they occurred) did indeed exist when they occurred.[12] Of course, we can say that when they occurred they existed now, namely at that previous time, and this is the sense in which presentism is true. Another way of saying this, then, is that everything that occurs indeed occurs now, but the now is the time of that occurrence, not the time at which we are talking about it. We must distinguish *the now that is the time of the event's occurrence* from *the now at which we are considering it*. To summarize: the existence of things in time is their existence at those times, not their existence now (if 'now' is understood as the time at which we are considering their existence). This is a hugely important point.

To recap the argument so far: there is more than one sense in which time may be said to be unreal. In one sense, licensed by the Aristotelian tradition, and revived by Mach and Barbour, it might mean that it does not exist independently of things, but is a kind of concomitant of or abstraction from motion. But this is very different from its being unreal in the sense that passage and change are unreal. This would have been flatly denied by the likes of Aristotle and Ockham, despite their argument for time's unreality from the unreality of its parts. Further analysis of that argument revealed that it depends on an ambiguity in the word "exists", or its synonym, "is real": the past and future are unreal only in the sense that they do not exist now, where 'now' denotes the time at which we are considering this. They nonetheless exist or are real at the times of their occurrence if they do indeed occur at those times. (Alternatively, we can say there is an ambiguity in the word 'now', depending on whether it refers to the time of the event's occurrence or the time at which this is being considered.) The same ambiguity was seen to infect the presentist axiom that gives this argument its apparent cogency, namely the premise that all (and only) things that exist now are real. If 'now' is taken as the time of these events' occurrence, then it reduces to a truism, whereas if 'now' is taken as the time at which the statement is uttered, it is merely false.

Still, one might insist, when we say past motions existed at past times, or that future ones will exist at future times, doesn't this presuppose the existence of the past and future times at which those motions must exist? If so, this becomes another motivation for adopting a static view in

[12] The same point is made by Adolf Grünbaum in his criticism of Hobbes's claim that "the present only has a being in nature", a claim which "depends on a tacit invocation of *present* occurrence as a logically necessary condition for having being or existing", so that Hobbes's claim reduces to "the mere tautology of 'only what exists now does indeed exist now'". (Grünbaum, 1971, 205).

which the now is excluded. If past events exist at past times, and future ones at future times, then all events alike have been presupposed to exist, and there is no place for temporal becoming. In this form, the static view is usually called *eternalism*. It is the view that all events and their temporal (and spatiotemporal) relations exist eternally, so that their coming into existence is precluded.

There is certainly an innocuous sense in which all the events and temporal relations in spacetime can be said to exist. The contrast here would be with a view that denied the existence of the material universe and the processes in it. But we must be very wary of using 'exist' in a temporal sense here, as advocates of eternalism do in their less guarded moments when they talk of future events being "already" real, and so not needing to become. Leibniz had the right response to the question of the existence of times when he wrote in an early dialogue: "Time itself ought not to be said to exist or not to exist at some time otherwise time would be needed for time." (A VI iii 565). He made a similar point in his criticism of Samuel Clarke in the last year of his life, when he objected to Newton's ally's describing duration as "immutable and eternal" (Clarke, 4th Letter, §10). "It cannot be said that a certain duration is eternal," wrote Leibniz, "but that the things which continue always are eternal, by gaining always a new duration." (To Clarke, V, §49: GP VII, 402). Just as a time cannot be said to exist at a time, so a duration cannot be said to exist at all times. Duration and time are not existents in the sense that they exist at times.

One might claim that temporal relations are eternal in the sense of existing timelessly. This would be in keeping with philosophical and theological tradition, where things that do not exist in time—numbers, for example, or God—have been called eternal. Such claims, however, are by no means unproblematic. God, for example, is supposed to act, and a timeless being can hardly be said to act.[13] This is perhaps why Newton, in opposition to theological tradition, identified God's eternity with his existence through infinite time, what had traditionally been called sempiternity.[14] But whatever difficulties may attend these traditional conceptions, it is not helpful to regard time and temporal relations as eternal, if only because the eternal—that which exists at no time—tends

[13] William Kneale, Sorabji p. 136.

[14] This view is most closely associated with Boethius, *De Trinitate*, 4, ll. 64-77. See Sorabji 1983, 116, and for discussion, Savitt Certainly, it was extremely important for Newton that God could be said to exist in time, to intervene in the world's operations. This was one of the main points of contention in his and Clarke's controversy with Leibniz, who took the view that (setting aside miracles) God would act through the actions of his creatures, in keeping with his omniscient anticipation of all their actions.

to get confused with the sempiternal or everlasting—that which exists at all times.[15] Finally, even if we regard *temporal relations* as existing eternally (timelessly), we cannot so regard *events*. *Events exist neither at no time nor at all times, but at the time of their occurrence*. Eternalism, it seems, is no more viable an alternative than presentism.

Thus the first major source of difficulties about time can be seen to lie in a lack of clarity concerning the senses of existence and reality. This afflicts the traditional argument for time's unreality from the unreality of its parts. Like its *presentist* premise, that argument was seen to be vitiated by a failure to distinguish between 'exists' or 'is real' in the sense of really occurring at the time of its occurrence, and 'exists now', i.e. exists at the time of that utterance. The rival *eternalist* view in a sense commits the opposite mistake, that of taking events and their temporal (and spatiotemporal) relations to exist eternally in the sense of *existing at all times*. But time and temporal relations should not be said to exist in time, while events exist at the time of their occurrence, not at all times. Thus if we keep firmly in mind what kind of existence we are attributing to things we can avoid these snares, and avoid this alleged dichotomy altogether.[16]

3. McTaggart and the "moving now" conception of passage

A second major source of difficulties concerning the status of the 'now' is that passage seems to involve events' somehow changing their status with respect to the 'now'. Those events which yesterday we regarded as happening now, we do not regard as now today. As Aristotle indicated, there are two ways to describe this state of affairs, but both seem to lead to paradox. One way is to admit that 'now' is changing, always different from one time to another. However, this seems to require that the 'now' itself must move along the time axis from an earlier to a later time. This is not a coherent notion if there is only one dimension of time.[17] But the other alternative offered by

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[15] Even Leibniz is guilty of this confusion, when he magnanimously allows in his criticism of Newton and Clarke that "if by saying that the duration of a thing is eternal it is only meant that the thing endures eternally, I have nothing to say against it", and that a thing's enduring eternally is to be understood as its "gaining always a new duration". But on Leibniz's own conception of eternity, God is eternal only in the timeless sense. "Gaining a new duration" is in keeping with Newton's equating of God's eternity with sempiternity, but not with timelessness.

[16] I give further discussion of the false dichotomy between presentism and eternalism in chapter 3, in the context of relativity theory.

[17] We should note that some have bitten that bullet, and have appealed to a second dimension of time to avoid this objection. Thus Mundle, Dobbs. Some even admitted the infinite regress, supposing an infinite hierarchy of times. But if one believes, as I do, that the premises of this argument are confused, this removes the main motivation for introducing additional time dimensions. Interestingly, despite its postulation of 11 or more spatial dimensions, String Theory works with only one time dimension. [or: The positing of precisely such a regress was one of the earliest responses to the McTaggart's arguments. There have also been attempts to argue for a two-dimensional time on

Aristotle is equally problematic, and this is to regard the now as "always the same thing". This suggests a presentist scenario in which "we" remain rooted in the now, and future events come gradually closer until they come to be now (and are therefore experienced by us), and then recede ever further into the past. Of course, it is no more possible for events to move with respect to the now than it is for the now itself to move with respect to events. Either way it appears that events must somehow change their relation to the now.

This was the starting point of the idealist philosopher John McTaggart Ellis McTaggart when he offered his famous argument for time's unreality near the beginning of the twentieth century. He began by imagining that all events, past, present and future, are given in their temporal relations: for instance, the partitioning of India precedes the assassination of Ghandi, the publication of the Communist Manifesto occurs after the births of Marx and Engels, and so forth. More precisely, he began by assuming that the "series of positions in time which runs from earlier to later" is given, a series he dubbed "the B series", and that "the contents of a position in time are called events" (458). He also assumed another series, which he called "the A series", the "series of positions running from the past to the present, and then from the present to the near future and the far future". Relative positions in the B series, he claimed, "are permanent, while those of the latter are not". He explained, "if M is ever earlier than N, it is always earlier than N. But an event, which is now present, was future and will be past." McTaggart calls each position in time a *moment*. Thus the relative positions of events in the B series, its moments, are permanent, but these moments change their (absolute) positions in the A series, from future to present to past. McTaggart grants that the distinctions of positions in the B series, being permanent, "might be held to be more objective, and to be more essential to the nature of time" (458). But he regards "the distinction of past, present and future" to be not only "as essential to time as the distinction of earlier and later", but, "in a certain sense … more fundamental" (ibid.) This is because the positions in the B series are permanent: there is no change there. But without change there cannot be any time. And change is only found, on this analysis, in the changing positions of the events in the A series. In the more famous and much analysed part of his argument, McTaggart then proceeds to find a contradiction in the idea that the moments (positions) in the A series can in fact change.

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similar grounds. On the opposite extreme, despite the proliferation of spatial dimensions posited by modern String Theory, proponents like Brian Greene have been firm in rejecting more than one dimension of time.]

Although reactions to McTaggart's argument have been many and varied, modern philosophers of science have tended to accept the basic structure of his argument. They have assimilated his critique of the A series to their own criticisms of the moving now. If events are changing their A-determinations, i.e. changing from future, to present to past, then their positions in the A series (their moments, in McTaggart's terminology) are changing. But any such changes will have to occur in time. Now moments cannot change in time (events cannot change their positions in time) without presupposing another time in which that change occurs. So, many philosophers have argued, the whole idea of events coming to be by taking on differing determinations of future, present and past must be abandoned. To suppose that events can change their positions relative to the now is just as contradictory as to suppose that the now itself can move. Concerning the first part of McTaggart's argument, though, where he claims that without change there can be no time, these philosophers point out that change is one thing, becoming another. Just as the temperature of a poker may be said to change along its length, so temporal change may be understood in terms of temporal difference: a velocity, for example, may be said to change over a given time interval if its value increases, say, across the interval, in the same way that the temperature of a poker may increase toward the end that is in the fire.[18] The B series, they conclude, is indeed "more objective, and … more essential to the nature of time", just as McTaggart had fleetingly suggested, and the A series is self-contradictory, making temporal becoming a mere illusion. This is the so-called "B theory" of time, usually assimilated to the static view.

In all this, however, there is a failure to recognize just how precarious are the foundations on which McTaggart's structure is built. If something is to be *permanent*, it must stay the same over time. It can only be true that event M "is always earlier than N" if there is a time in which the temporal relations between these events could conceivably change. But this is to assume a time outside time. Another way of putting this is as follows. McTaggart's initial supposition that events are "given" in their (B series) temporal relations is not as innocent as it may seem. One can imagine all events and their temporal relations *sub specie aeternitatis*, as they used to say, "from the point of view of eternity". But this is to imagine them as existing timelessly. If you now introduce time, an event exists where and when it occurs. It does not exist at the time you are considering it just because at that time you can conceive it as existing. And it does not exist (together with its

[18] See for example Grünbaum, Smart. {*Find out where this analogy with the poker occurs.*}

relations to other events) at every time; that is a surreptitious reinterpretation of "eternal" as "existing through time". As we saw above, time and spacetime do not exist through time.

What this means is that the B theory perspective, the "static" view, is just as problematical as the A-theoretic view. The A theory assumes that events can change their temporal relations, the B theory insists that these relations remain the same. But both views illicitly presuppose a time during which the relations either change or remain permanent. Both perspectives begin by assuming that all events "already" exist, and ask what more is required. The A theorist says they must become 'now' at some time; the B theorist says that if they already exist, there is no need for them also to become. But the fallacy lies in interpreting the existence involved in assuming that "there are events" as a temporal existence. It is not. There are not future or past events in the sense that they exist at the time of my saying this. *The existence of things in time is their existence at the times of their occurrence.*

What, then, are we to make of the notion of the flow of time? Are we to abandon it as irredeemably infected with contradiction? Well, certainly the model of time flow in which there is a relative motion of events relative to the now should be given up, as I have argued above. For whether it is the events that are fixed and the now that is moving, or the now that is fixed while future events move up through it and into the past, such changes will presuppose another time dimension. But what about a more innocent/less contrived conception, in which the flow of time is modelled directly on motion, on that of a point moving along a line from left to right, say? For surely if we say that passage from earlier to later is an illusion, then consistency demands that we should also deny that there is any flow of the moving point from left to right.

In fact there are classical precedents for such a modelling of time flow, from Aristotle to Newton. An interesting example is provided by the now mostly forgotten medieval thinker, Peter Aureol. Convinced by the classical argument for time's unreality with which we began this chapter, Aureol conceded that "the past and future, between which the indivisibles establish continuity, have no being if the mind does not conceive them." ({*reference*}). Nevertheless, he was able to call on the authority of Averröes to contend that there is still something of time that *does* exist outside the mind, namely the indivisibles that establish this continuity: "whether the intellect

considers them or not, the indivisibles of time and of motion exist outside the mind".[19] From this Aureol constructs an early version of presentism. There is only one time, constituted by the flow of the objective now (*nunc*). But there is only ever one such now:

> Time is constituted by the flow of these nows from earlier to later. But it is not possible that there is more than one now. It is therefore impossible that there is more than one flow, more than one time. … According to what the mathematicians imagine, a flowing point engenders a line, as the now engenders time by flowing. But if there can exist only a single point, there can exist only a single flow from this point, and, consequently, only a single line. (Aureol, Duhem 1985, 302)

We won't pursue this presentist aspect of Aureol's thought, having dealt with the fallacy of the classical argument for time's unreality above. But the idea that time is generated by the flow of a moving now represents a tradition in the philosophy of time lasting from Aristotle to Newton. Aristotle appealed to the analogy with the generation of a continuous line by a moving point in his discussion of the continuity of time.

> A now follows a moving object, just as time follows change; for it is the moving object that enables us to know before and after in change… So time is not only continuous thanks to the now, but is also divided at the now, because this too follows the nature of the movement and the moving object. (Aristotle, *Physics*, IV, 219 b26-28, 220 a4-6).

As Aureol explains, "Wanting therefore to give the means by which we ought to conceive time, Aristotle takes as example the way the mathematician imagines the generation of the line by the flowing of a point." There is more to this than mere analogy. The comparison of the generation of time with the generation of the line exploits the close relationship between time and motion whereby each measures the other. As a line is drawn from left to right, the point of the pencil passes through all the points on the line, viewed as completed. Each point corresponds to an instant of the time (or 'now') during which the line was drawn. As Aristotle says, in this sense "the nows are always different". So there is no motion of a now. There is, however, passage, the tracing of the pencil from left to right. But if there is passage from one spatial end of a motion to the other during a given interval of time, why should there not also be transition from one temporal end of the motion to the other as the space is traversed? In this sense, the reality of passage stands or falls with the reality of motion.

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[19] He quotes Averröes in support: "The mind conceives all the parts and posits them as existing, at the same time that it conceives the indivisible that exists in reality." ().

A large part of the appeal of the flux of time lies in the geometrical tradition, perhaps going back to Archytas of Tarentum, of lines being generated in time by the flow of a point. In this way, it was felt, a line could be generated without having to conceive it as composed of points—a notion that was well known to lead to paradox. The continuity of the line is guaranteed by its being generated by a continuous motion in a continuous time. Newton's motivation was similar. Having been shown by his teacher Barrow the mathematical utility of generating figures by motion for the solution of tangent problems, he abandoned his earlier flirtation with Epicurus's time-atoms in favour of a generation of time by a continual flux:

> I don't consider Mathematical Quantities as composed of Parts *extreamly small*, but as *generated by a continual motion*. Lines are describ'd, and by describing are generated, not by any apposition of Parts, but by a continual motion of Points. Surfaces are generated by the motion of Lines, Solids by the motion of Surfaces, Angles by the Rotation of their Legs, Time by a continual flux, and so in the rest. These *Geneses* are founded upon Nature, and are every Day seen in the motion of Bodies. (Newton 1964, 141)

What are we to make of this analogy? Taken literally, the idea of time flowing seems irreparably flawed. As many critics have asked, if it flows, how fast does it flow? The very question seems to show the absurdity of the notion, since time is what is used to measure the speed of anything. On the other hand, though, time is measured by motion, and not just any motion, but an "equable" (i.e. uniform) one. As Newton saw, any time whose measure is given by a particular motion taken to be uniform (such as that of a pendulum clock or the eclipses of the moons of Jupiter) is only a *relative time*. But this raises the prospect that the time beaten out by one such regular motion, say that of the Jovian eclipses, would appear irregular when compared against the periodicity of another, say the pendulum. So relative times, at any rate, would not necessarily flow in unison; yet it would seem a matter of convention which of them is to be taken as flowing uniformly. To Newton this was anathema, and he proposed the equable flux of absolute time as the solution. As Barbour has explained, by requiring this time to be the one whose equation was determined by astronomers, he "demonstrated the possibility of introducing an essentially unique time parameter with respect to which innumerable different motions could simultaneously be made to obey basically the same laws of motion" (633). The supposition of an "equable flux of time" then is equivalent to the supposition that there are uniform motions (even if nothing actually moves completely uniformly). That the flow of absolute time is uniform is, perhaps, a convention. But the

supposition that there is one such flow is not. It is a mark of what Barbour has called the Phenomena marching in step. The hypothesis of absolute time, of course, is refuted by the success of relativity theory, as we see below. But the correlation of time with motion again opens up the possibility that time flows at different rates for processes taking different paths through spacetime. Even so, these differing rates of flow are related to one another in a non-conventional way, preserving this aspect of Newton's insight.

In sum, the objections to the passage of time raised by critics of the "moving now" conception of passage seem to have force only against this conception, rather than against the reality of passage itself. In particular, the idea that passage involves the changing of the relation of events to the 'now' is presupposed by McTaggart in his famous argument for the unreality of time, and consequently underlies both the 'A' an 'B' theories. The A theory assumes that events can change their temporal relations, the B theory insists that these relations remain the same. Both views illicitly presuppose a time during which the relations either change or remain permanent. We saw that temporal relations can perhaps be said to exist timelessly, but events cannot: each event exists at exactly the time when it occurs, and not at other times.

We then examined the notion of the flow of time. Even though the conception of this as the motion of a now is flawed, as critics have suggested, it was seen that this in itself is not sufficient to refute the idea of passage. The traditional model of passage involved the comparison of the flux of time with the generation of a line from a moving point. Here the passage from an earlier 'now' to a later one is modelled by the passage of a point moving from the left end to the right end of a line segment. This seems to be free of the difficulties usually levelled against passage: there is as correspondence between the earlier 'now' and where the point is in the earlier part of its motion, and the later 'now' and where the point is in the later part of its motion. The 'now' does not itself move. But if the point moves, it passes through a stretch of space over an interval of time, from the earlier time to the later. If this passage from an earlier to a later time is denied, motion, too, must be denied.[20]

[20] Barbour, to his credit, realizes this. In his vision, "There is simply no motion or change at all." (199, 264). But most physicists will not follow him here, as he ruefully acknowledges.

4. Russell and the "at-at theory" of motion and passage

We now turn to a third set of objections to the reality of passage, stemming precisely from such a linear model of passage based on the analogy with motion. This has its origins in the famous Arrow Paradox of Zeno of Elea. Plato testifies that Zeno devised his paradoxes to pay back in kind the critics of his teacher and lover Parmenides, who held that reality was changeless, and motion but a deceptive appearance. According to Aristotle, in the third of his paradoxes designed to refute the reality of motion, Zeno claimed that

> If it is always true that a thing is at rest when it is opposite something equal to itself, and if a moving object is always in the 'now', then a moving arrow is motionless. (*Physics* VI, 9: 239 b5-8)

In a more modern idiom, this might be re-expressed as follows: Granted that there cannot be motion in an instant, then the arrow is not moving at any instant of its flight; therefore it does not move at all. Aristotle dismisses this paradox as a fallacy of composition: "Here the conclusion depends on assuming that time is composed of nows; if this assumption is not granted, the argument fails." (*Physics* VI, 9: 239 b29-32). Elsewhere in the same chapter he says more. An indivisible for him is an endpoint of a magnitude or interval, a *now* is the endpoint or beginning of a stretch of time. There can be motion across a stretch, but not at an instant. Thus for Aristotle "nothing moves in the now" (*Physics* 234a 24); but by the same token, "nothing can be at rest in the now" either (*Physics* 234b 7), since motion and rest both require an interval of time. So he can cede to Zeno that the arrow is not moving at any and every now without at all conceding that it is motionless. One curious consequence of Aristotle's view is that even though at any instant of its motion Zeno's arrow is not moving, it will nevertheless be true to say at any instant of the motion that the moving arrow *has moved*. This view is curious, but it is not absurd.

When we look at this paradox in the light of the *Differential Calculus*, however, it seems that we can say something more satisfactory. The difference between the arrow at rest and the arrow in motion is that the latter has a *non-zero instantaneous velocity* (and perhaps, non-zero acceleration too). This accords with our intuition that the arrow, so long as it is in flight, has a tendency to keep on moving. Galileo attempted to capture this intuition with his notion of *degree of speed*, as did Descartes with his insight that a moving body has a *state of motion* at each instant, together with a *conatus* or endeavour to move. Leibniz generalized the idea of *conatus* into his notion of *appetition*, the tendency of each substance at every instant to change its state. But above all Newton was responsible for laying the foundations of the modern approach with his idea that every magnitude

is a *fluent*, a quantity varying in time, which has a well defined rate of change or *fluxion* at each instant of time. Thus at time t, the fluxion of the displacement $s(t)$ of the arrow from its starting point is $v(t) = s^{\cdot}$, or, in Leibniz's more familiar notation, $v(t) = ds/dt$. If the arrow is at rest, $v(t) = 0$, if it is moving, $v(t) \neq 0$. In both Leibniz's and Newton's understandings of the calculus, ds/dt represents the limit of a sequence of finite ratios $\Delta s/\Delta t$, where Δs is the increment of the displacement $s(t + \Delta t) - s(t)$ achieved in the increment of time Δt, as Δt becomes arbitrarily small.

Rather surprisingly, however, Bertrand Russell summarily dismissed any such attempt to resolve Zeno's Arrow Paradox by appealing to the calculus. Expressing himself with characteristic brio, he vehemently rejected the idea of the moving arrow's having a state of motion at each instant:

> People used to think that when a thing changes, it must be in a state of change, and that when a thing moves, it is in a state of motion. This is now known to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another. (1929, 83-84)

> "If everything is in rest or in motion in a space equal to itself, and if what moves is always in the instant, the arrow in flight is immovable." This has usually been thought so monstrous a paradox as scarcely to deserve serious discussion. To my mind, I must confess, it seems a very plain statement of a very elementary fact, and its neglect has, I think, caused the quagmire in which the philosophy of change has long been immersed. In Part VII, I shall set forth a theory of change which may be called *static*, since it allows the justice of Zeno's remark. (1903, 350)

In these passages Russell alludes to the theory of change and motion that he outlines in Part VII of his *Principles of Mathematics* (Russell 1903). According to this analysis, now widely accepted by contemporary philosophers of science under the moniker the *at-at theory of motion*, all there is to motion is an occupation of different places at different times: "when different times, throughout any period however short, are correlated with different places, there is motion; when different times, throughout any period however short, are all correlated with the same place, there is rest" (1903, 473). And motion consists "*merely* in the occupation of different places at different times, subject to continuity … There is no transition from place to place" (473). Russell's denial here of any transition or passage from place to place explains why he calls his analysis a *static* theory of change.

Russell's at-at theory has been highly influential, especially in the form in which it was promulgated by Wesley Salmon, and its adoption by philosophers such as Adolf Grünbaum and Jack Smart has done much to cement the "B-theory" or "static view" as the dominant philosophy

of time among scientifically minded philosophers. There are, however, some troubling ambiguities in Russell's own statement of the theory, and these ambiguities, though subtly transformed, continue to bedevil contemporary statements of the static view. For example, in a further passage from his (1903) reprinted in his later *Mysticism and Logic*, Russell says

> Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest. The only point where Zeno probably erred was in inferring (if he did infer) that, because there is no change, therefore the world must be in the same state at one time as at another. (1903, 347; 1929, 80-81)

In this passage Russell explicitly denies change and motion—we'll come back to his justification of this by appeal to Weierstraß in a moment—whereas in the other quoted passages he is only denying an instantaneous state of motion, not motion and change themselves. Here he claims the arrow is at rest at every moment of its flight, whereas according to the analysis in Part VII of the *Principles* the arrow would only be at rest if at the different times of its flight it were at the same place. The way Smart and Grünbaum resolve this ambiguity is to disown Russell's denial of the reality of change or motion. On their interpretation of it, the at-at theory is rather to be understood as construing change as *temporal difference*, while denying that there is any "transition" from one state or event to another.[21] There is motion from point A to point B if a body is at A at an earlier time, and at B at a later time. Thus there is no necessity to deny motion, nor does either of these authors want to deny it. But there is no transition, in that there is no becoming at any instant: each event in the arrow's flight "tenselessly occurs" at the time of its occurrence.

Thus Zeno's claim that the arrow is at rest at every instant of its flight (and Russell's reckless endorsement of it) is rejected on the more sober interpretation of the at-at theory (the official version he gives in *Principles,* Part VII): the arrow can only be said to be at rest or in motion *across an interval of time*, for only then can the correspondence demanded by the at-at theory be established between the places it is at and the times it is at them. In this respect, the sober version of the at-at theory is in complete conformity with Aristotle's position discussed above. It is no more true on this view that the arrow is at rest at each instant than it was on Aristotle's. And to infer the unreality of motion from the fact that there is no motion at each instant is the very fallacy

[21] This is perhaps implicit in Russell's remark that "change is due, ultimately, to the fact that "many terms have relations to some parts of time that they do not have to others. But every term is eternal, timeless, and immutable…" (1903, 471).

of composition pointed out by Aristotle, just as invalid on the sober at-at view as it was on his. It will even be true on the at-at theory, as it was on Aristotle's, that whilst the arrow can never be said to be *moving now*, it can be said to *have moved* at any instant of its motion. The difference is that on the Smart-Grünbaum interpretation, Zeno's claim that the arrow is at rest at each instant is replaced by the claim that there is no passage or becoming in an instant.

But here is an interesting point. It is no more valid to infer from this that *there is no passage or transition across an interval of time* than it is to infer from the fact that there is no motion at each instant that *there is no motion across an interval of time*! According to the at-at theory, a motion can take place over an interval even if there is never motion at any instant; similarly, process or becoming could take place over an interval even if there is never any transition or becoming in an instant. Russell's inference from the at-at theory to a "static" theory that denies change and motion is invalid; but then so equally is Grünbaum's and Smart's inference from the at-at theory to a "static" theory that denies passage—the "becomingless" theory, as Grünbaum calls it.

But why did Russell deny that a moving body is in a state of change at each instant, and what motivated Grünbaum to be so hostile to becoming as to assert "the objective *becominglessness* of physical events" (Grünbaum 1971, 214)? In order to understand these things, we have to dig a little deeper into the origins of their views. Russell's deep antipathy to passage stems from his having made acquaintance with the new mathematics of continuity emanating from continental Europe in the late nineteenth century at the same time as philosophers were using an earlier understanding of continuity to argue for the *a priori* generation of continuous magnitudes from infinitesimals. The immediate source of his displeasure was the philosophy of Hermann Cohen, whose *Das Prinzip der Infinitesimalmethode* (1883) had become the blueprint for the influential Marburg School of neo-Kantianism.[22] On Cohen's view, objects are necessarily represented as having determinate spatial and temporal locations: their generation from infinitesimal intensive magnitudes is the condition for the very possibility of their being represented as real objects.[23] This involved a reinterpretation of Kant's Anticipations of Perception, according to which the infinitesimal is an *intensive magnitude*

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[22] See Monier-Williams 2007. It was in opposition to Cohen's neo-Kantianism that both the logical positivism of Rudolf Carnap and the phenomenology of Martin Heidegger were conceived. See Friedman 2000.

[23] Cf. Cohen, p. 144: "*This presupposition* [of intensive magnitude] *is the meaning of reality and the secret of the concept of the differential*"; quoted from Russell 1903, p. 344.

from which *extensive magnitudes* such as space, time and velocity are generated *a priori*.[24] For instance, velocity is generated from its infinitesimal *dv*, time from the infinitesimal moment, *dt*. In this way continuity is implicated in the very generation of the *a priori* laws that are formal conditions of the possibility of experience of objects.

Russell, newly convinced by Cantor of the impossibility of infinitesimals, regarded Cohen's notion that the continuum is generated from infinitesimal elements with unrelenting scorn. Moreover, Cohen's welding of Kantian *aprioism* to this outdated conception of continuity was a red flag in the face of the bullish young anti-idealist. Fortified by the new theories of Weierstraß, Dedekind and Cantor, the three men whom he credits with having "completely solved … the problems of the infinitesimal, the infinite and continuity" (1929, 81), Russell simply lowered his head and charged:

> Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest. … In the case of motion, [Zeno's argument] denies that there is such a thing as a *state* of motion. In the general case of a continuous variable, it may be taken as denying actual infinitesimals. For infinitesimals are an attempt to extend to the *values* of a variable the variability which belongs to it alone. When once it is firmly realized that all the values of a variable are constants, it becomes easy to see, by taking any two such values, that their difference is always finite, and hence that there are no infinitesimal differences. … This static theory of the variable is due to the mathematicians, and its absence in Zeno's day led him to suppose that continuous change was impossible without a state of change, which involves infinitesimals… (347, 351-2)

There is much to say about this passage. First, from the fact that Weierstrass can define continuity without appeal to actual infinitesimals, it does not follow that you cannot define continuity by appeal to infinitesimals. In fact, the refutation of infinitesimals by Cantor on which Russell relied was not sound. There were perfectly good theories of non-Archimedean infinitesimals even when Russell wrote this (long before Abraham Robinson, but in differential geometry rather than in analysis), and there have been ever since. Thus, as a matter of historical fact, Weierstrass did not "banish infinitesimals", as Cantor and Russell so badly wanted to believe, except (for some decades) from analysis. Secondly, modern theories of actual (i.e. non-Archimedean) infinitesimals

[24] Immanuel Kant, *Critique of Pure Reason* (Kant 1929, 200 & ff): "In all appearances, sensation, and the *real* that is an object of sensation has *intensive magnitude*, that is, a degree." (B 207)

do not necessarily conceive them as variables.[25] But the sense in which they are "static" is a mathematical one, not a metaphysical or physical one, involving a lack of motion or change.[26] Thus Russell's argument from the mathematics of the continuum to metaphysics is completely misplaced.

But all this is in any case beside the point. One does not have to be committed to the existence of infinitesimals in the continuum to believe that a derivative—such as *ds*/*dt*—is not only mathematically well-formed, but has a correlate in physical reality. The state of change depends only on the existence of time-derivatives, and does not stand or fall with the existence of infinitesimals in the continuum. Russell has a very curious stand on derivatives, portraying them as mere numbers without physical correlates:

> It is to be observed that, in consequence of the denial of the infinitesimal, and in consequence of the allied purely technical view of the derivative of a function, we must entirely reject the notion of a *state* of motion. Motion consists *merely* in the occupation of different places at different times, subject to continuity as explained in Part V. There is no transition from place to place, no consecutive moment or next position, no such thing as velocity except in the sense of a real number which is the limit of a certain set of quotients. The rejection of velocity and acceleration as physical facts (i.e. as properties belonging *at each instant* to a moving point, and not merely real numbers expressing limits of certain ratios) involves, as we shall see, some difficulties in the statement of the laws of motion… (Russell 1903, 473)

To say that this "involves some difficulties" is not so much an understatement as a reduction of Russell's position to absurdity: velocity and acceleration *are* physical facts, not "merely real numbers expressing limits of certain ratios".

But not all Russell's criticisms of infinitesimalist theories like Cohen's are unjust. One of them, mentioned in the just quoted passage, concerns the assumption of successiveness: If one assumes (as Cohen appears to) that Zeno's arrow progresses from point x to its next position at point $x + dx$ by the accession of the infinitesimal element dx, then this is incompatible with the *denseness* property of the continuum: the property that, between any two points in the continuum, there is a further point. In a word, there is *no next point in the continuum*. But as Russell realized, this is fatal to

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[25] Category Theory, however, has rehabilitated the idea of the variable quantity; and on this basis John Bell, among others, has erected a modern theory of infinitesimals, Smooth Infinitesimal Analysis. See

[26] In such theories there are infinitesimal neighbourhoods of points, but no two assignable points are separated by an infinitesimal distance. In the passage just quoted Russell simply assumes the Archimedean property as a premise in an argument against non-Archimedean elements in the continuum.

intuitions about how things continually come into existence, since it is usually assumed that any given event comes into being out of the preceding one. Without this there seems to be a gap in causality.

Opponents of Russell seized on this "gappy" character of the Cantorian continuum. Thus William James opposed it with "the more empirical or perceptual notion that anything is continuous when its parts appear as *immediate next neighbors, with absolutely nothing between*." Likewise, Alfred North Whitehead—Russell's collaborator on the monumental *Principia Mathematica*, and author of a rival theory of gravity to Einstein's with a flat background space—proposed a *pulsational theory of becoming* based on sense experience. He ceded Russell's criticism: "every act of becoming must have an immediate successor, if we admit that something becomes. For otherwise we cannot point out what creature becomes as we enter upon the second in question…" (Whitehead 1929, 107). Consequently he proposed that the creatures which become are "actual occasions" that come into being discretely, and that any such "*actual entity is an act of experience*" (105). Even though the actual occasions are discrete experiential acts, they "constitute a continuously extensive world": "There is a becoming of continuity but no continuity of becoming" (53).

Grünbaum considered that Russell had opened the door to such sensation-based theories by insisting that the relation of succession must be given in experience. According to him, "the very meaning of sensed becoming or occurring-later presupposes (i.e. entails) 'nextness'… If the 'later-than' relation is defined intuitively, it is clear that no dense temporal order of events can be created by an ordering relation whose very meaning involves 'nextness' and hence excludes denseness" (Grünbaum, 1950, 165). Thus having defined the relation of succession as given in sensation, Russell was not entitled to postulate the property of denseness for the ordering of successive instants, and therefore not entitled to presume that they would form a Cantorian continuum.[27] Grünbaum believed that this difficulty in Russell's position could be obviated by adopting an ontology of *point-events*—by "postulating point-instant-particles, i.e. *events*, rather than things, to be the basic entities of nature" (1950, 152). These point-events, unlike sensed events, could be

[27] Having followed Whitehead in defining instants by the method of extensive abstraction from the extended events of experience, "Russell gratuitously and falsely assumes that a Cantorean theory of becoming can be made intelligible and convincing on the sensationalist assumptions of his program." (Grünbaum 1950, 186).

assumed to exist in one-one correspondence with the points in a Cantorian continuum.[28] Such events, he claimed, "simply are or occur (irreversibly in some cases) but they do not "advance" into a pre-existing frame called 'time'." (1950, 172) Similarly, Jack Smart insisted that "Events do not come into existence, they occur or happen." (Smart 1949, 486). Thus, these authors are arguing, since there is no coming into existence of point-events, and all the point-events in a continuous process can be put in a in one-one correspondence with the points in a Cantorian continuum, there is no becoming or passage in any process.[29]

Now we have already exposed the fallacy above of inferring a lack of passage across an interval from a lack of passage at any instant. Motion occurs across an interval even though there is no movement from one point to the next, if these points constitute a dense series; so there is no reason passage cannot occur through an interval despite the denseness of point-events. Even if there can be no becoming of one such event out of its immediate predecessor, as Whitehead and James required of becoming, this does not entail that such a point-event cannot have become. (This is the analogue of what was said above in connection with both Aristotle and the at-at theory: at any 'now', the arrow can be said to have moved, even if on those theories it cannot be said that it is moving at each 'now'.) Provided a point-event is the terminus of a process, there is a passage from earlier events to it, and it can therefore be said to have become.

But actually there is a deeper lying objection to the at-at theory itself. This is its founding assumption that "motion consists *merely* in the occupation of different places at different times, subject to continuity", that is, the assumption that the existence of a one-one correspondence between the point-events in a continuous process and the instants in a temporal continuum is sufficient for motion.[30] Russell, as we saw, bit the bullet and rejected velocity and acceleration as physical facts. But an instantaneous state of motion is most definitely to be physically distinguished from an instantaneous state of rest; and since they are not so distinguished on the at-at theory, it is

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[28] "Clearly", Grünbaum wrote, "these events are not sensed and their properties differ fundamentally from such perceived 'events' as the sensed coincidence of thunder and lightning. … *the events constitute a linear Cantorean continuum with respect to the relation 'later-than'! In this way the concept of 'later-than' becomes the key to the temporal order without involving the nextness property*." (1950, 152, 168-9)

[29] "*Since individual events do not move*, but are nevertheless ordered non-consecutively by 'later-than', it is the theory of events as here interpreted which justifies Russell's writing: 'Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world…'". (Grünbaum 1950, 179)

[30] Cf. Russell: "In the first place, time and space may be replaced by a one-dimensional and *n*-dimensional series respectively. …If we add that the one-dimensional and the three- dimensional series are to be both continuous, and that each many-one relation is to define a continuous function, then we have all the kinematical conditions for a system of material particles, generalized and expressed in terms of logical constants." (Russell 1903, 468).

therefore inadequate for modern physics.[31] The derivative is not a mere number, but is a vector quantity (more accurately, a one-form), and has directionality, unlike a mere spatial point at a time. Similar considerations apply to the ontology of point-events adopted by Grünbaum. He holds that "a linear physical movement consists of a Cantorean continuum of events each of which simply occurs", and that "we can give a physical process as a co-ordinating definition for a 'continuous set of instants'." (1950, pp. 154, 171). But if one asks whether a continuous manifold of point-events is sufficient for representing motion, the answer is quite clearly, no. Processes in spacetime are not represented simply as aggregates of point-events: considerably more structure must be included in the mathematical model in order for it to represent physical processes, including the tangent space structure of differential geometry.

There is one final philosophical point that needs to be made concerning the understanding of 'now' presupposed by Smart and Grünbaum. This is their claim that although the "tenseless occurrence" they ascribe to point-events is perfectly objective, both becoming and the 'now' necessarily make reference to a perceiving subject. This distinction probably has its origin, as Grünbaum suggests, in Bertrand Russell's claim in his (1915, 212), that "past, present and future arise from time-relations of subject and object, while earlier and later arise from time-relations of object and object." (Grünbaum 1971, 215-216). Grünbaum amplifies this into the claim that the becoming of an event is simply its coming into someone's conscious awareness. But even though 'now' does contain an implicit reference to something extrinsic to the event, consciousness has nothing to do with it. The valid core of this idea of appears rather to be this: 'now' is an *indexical term*, like 'here', 'I' or 'this'. Indexical terms pick out different objects or places or times depending on the context in which they are uttered.[32] So if I am present when an event occurs, I will say it is happening *now*, meaning, "at the time of my utterance". But the time of my utterance is something entirely objective, even if the utterance involves me as subject. It will usually be the case, of course, that the time of my utterance is no business of physics—but not necessarily so. For example, a standard unit of measurement in cosmology is 'bya', standing for billions of years ago, meaning billions of years before *now*, before the present era. Earth is thought to have formed, for instance, not long after 5 bya, and the Big Bang is thought to have occurred about 14 bya. As a result,

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[31] This is relevant to discussions of time-reversal, where one has to reverse the direction of the instantaneous velocity at each point (transform each state into its t-inverse) in order to perform time reversal, as Earman argued in an early paper.

[32] The expression 'indexical' was introduced by the American philosopher Charles Sanders Peirce,

textbooks on cosmology abound with diagrams where *now* is clearly labelled, contrary to oft-repeated claims that it has no place in physics.[33] Granted, this is not an instantaneous now; but the point remains that the 'now' in question is indexical, picking out the time at which we intelligent observers are constructing these theories of origin. But it is also perfectly objective: the occurrence of the Big Bang 14 billion years before now is an observable consequence of current theories which is empirically falsifiable.

The indexicality of the *now* relates to tense as follows. A tensed use of a verb gives implicit information about the time of utterance; a tenseless use does not. But tense is a characteristic of verbs, not of happening itself, as the phrase "tenseless occurrence" would suggest. Whether a given event *a* occurs or not has nothing to do with whether it is registered in anyone's awareness. Grünbaum claims that the latter statement is true only if the word 'occurs' here is used tenselessly. But we can express that same statement using a tensed verb. If *a* is (tenselessly) earlier than my utterance of the last statement, then it will be equally true to say that the event *has become* (by the time of my uttering this) independently of anyone's experiencing it; and if I utter this before *a*, I would say that the event *a*, if it occurs, *will occur later* than my utterance independently of anyone's experiencing it. Thus Grünbaum's claim that whereas "events happen tenselessly" (1971, 215) "becoming … requires the occurrence of certain *conceptualized conscious experiences* of the occurrence of physical events" (1971, 197) seems unfounded. An event occurs, happens or becomes exactly when it occurs, happens or becomes, independently of any minds or clocks. If we say an event OCCURS, using the verb 'occurs' tenselessly, then this describes the way we have used the verb, not a variant kind of existence or occurrence.

So none of the third set of classical objections to passage relating to its absence in the now has hit its mark. Russell's attempt to infer a static theory of change from the Weierstaß-Dedekind-Cantor mathematical model of the continuum was seen to involve several errors. The values of a variable are not constant in the sense that they remain unchanging through time, so that it is a category mistake to enlist Weierstaß's static theory of the variable in support of the idea that reality is unchanging. Meanwhile Cantor's proof that there are no infinitesimals in the continuum is not sound as Russell had supposed, since there are perfectly consistent theories of the continuum involving non-Archimedean infinitesimals. In any case, Russell's assumption that the idea of a state of change stands or falls with infinitesimals cannot be accepted, since the derivatives characterizing

[33] This denial of the now in physics can be quite ironic. In his *About Time*, Paul Davies

an instantaneous state of change are regarded as perfectly well defined without need of an appeal to infinitesimals. Again, Russell's and Grünbaum's inferences from the at-at theory to the impossibility of passage are themselves invalid: from the fact there is only a point event or instantaneous state (and no actual becoming) at each instant in an interval, it does not follow that there is no passage in that interval, any more than it follows from the fact that there is only an instantaneous velocity (and no actual motion) at each instant that there is no motion over the interval. Finally, we saw that *now* is correctly described as indexical, making implicit reference to the time of utterance, like tensed verbs, but that this neither makes it subjective, nor precludes it from having a role in physics. Likewise, the idea that there is a distinction between "tenseless occurrence" and becoming was seen to be an error. You can describe an event's occurrence using tense, or not, but this does not make for two kinds of occurrence. An event cannot exist or occur without having become. Once we have represented all events and all processes on a spacetime diagram, we have represented all becoming, so it is unreasonable to look for something else to be superadded.

This concludes our discussion of the classical problems concerning the 'now' and the reality of time and passage.
