Beeckman, Descartes and the force of motion

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Abstract

In this reassessment of Descartes' debt to his mentor Isaac Beeckman, I argue that they share the same basic conception of motion: the force of a body's motion—understood as the force of persisting in that motion, shorn of any connotations of internal cause—is conserved through God's direct action, is proportional to the speed and magnitude of the body, and is gained or lost only through collisions. I contend that this constitutes a fully coherent ontology of motion, original with Beeckman and consistent with his atomism, which, notwithstanding Descartes' own profoundly original contributions to the theory of motion, is basic to all his further work in natural philosophy.

1. Introduction: Descartes' Castigation of Beeckman

In October 1629 René Descartes reacted with scorn and indignation to suggestions that he might have learned anything from his good friend Isaac Beeckman during their enthusiastic collaboration in Breda in 1618-19.[1] Relations deteriorated, and a year later, in October 1630, Descartes wrote a long and vituperative letter to Beeckman that was clearly intended to be psychologically crushing, in which he scathingly disavowed any influence from Beeckman whatever.[2] This has been interpreted by Klaas van Berkel as an attempt by Descartes, and a successful one too, to discourage Beeckman from his intention to publish his own natural philosophy, the *Mathematico-physical Meditations*, a selection of writings from his *Journal*.[3] Had Beeckman done so, van Berkel observes, this would have been severely embarrassing to Descartes' claims to originality in his *Physics* (or *Monde*), the treatise on natural philosophy he was then writing and hoping soon to publish.[4] For not only was the general programme of explaining phenomena in terms of micro-mechanical methods much the same in both works, so were some of the particular mechanisms, for instance, the explanation of magnetism in terms of the action of Beeckman's proposed magnetic corpuscles.[5] It would also have been evident that Descartes' way of applying mathematics to natural phenomena was not derived by him from first principles, but learned by him from Beeckman. This would have been particularly obvious in the application of mathematics to music, and to the problem of falling bodies.

Now, in one sense this claim is uncontroversial. For since Cornélis de Waard discovered a copy of Beeckman's *Journal* (in 1905) and published it together with related material in the 1930's,[6] there has been a general acknowledgement of Descartes' profound debt to Beeckman in natural philosophy, not only for the micro-mechanical explanations of phenomena, but also specifically for his "principle of inertia" and the idea of conservation of motion. But these claims of influence have also been mitigated by a variety of considerations. On the one hand, it is pointed out that by 1630 Descartes had constructed a coherent program for the application of geometry to philosophy that went far beyond the isolated insights of Beeckman;[7] and on the other, that Descartes had not, in any case, fully grasped the principles of Beeckman's physics in 1618, which he rediscovered for himself only after many years' further labour. Thus Alexandre Koyré, in his celebrated study of the problem of fall, claims that in 1618 Descartes "simply throws away Beeckman's intellectual prize, the principle of conservation of *motion* [and] substitutes the conservation of *force*." (*Galileo*, 83). Koyré proposes, moreover, that Beeckman himself does not properly understand the true significance of this "intellectual prize", which is first made clear in Descartes' mature formulations of the geometrical character of motion and (what we call) the principle of inertia.[8] Now although much has been written on Descartes' natural philosophy since Koyré wrote in 1939, his defence of Descartes in the light of de Waard's revelations has remained highly influential. Accordingly, there has been a tendency to acknowledge Beeckman's contribution as a *source* of Cartesian physics, but to dismiss his contribution as relatively unoriginal and naïve.[9] Of course, if this is so, then the extraordinary vehemence of Descartes' attack on Beeckman in 1630 seems out of all proportion to any threat that Beeckman's publishing could hold, and therefore to require a more psychological explanation.[10]

I shall argue here that this standard construal profoundly underestimates the extent of Descartes' debt to Beeckman. I contend that there was no misunderstanding between them about the physics of fall in 1618, and that they conceived motion in fundamentally the same way, in terms of God's conserving a body's motion, together with a certain force of motion proportional to the speed and magnitude of the body, and shorn of any connotations of internal cause. That is, a body persists in its motion as a direct result of God's conserving it, just as it persists in its rest, until its motion is changed by collisions with others, in such a way that the overall force of motion is conserved. I argue that this conception is clearly spelled out by Beeckman in his *Journal* entries from before he met Descartes, whereas there is no record of Descartes' having obtained this conception independently. I argue further that this constitutes a fully coherent ontology of motion, original with Beeckman and consistent with his atomism, which, notwithstanding Descartes' own profoundly original contributions to the theory of motion concerning directionality and instantaneous tendencies to motion, is basic to all the latter's further work in natural philosophy. This being so—and notwithstanding the great advances that he would have correctly seen himself as having made—Descartes would have had good cause for profound anxiety about Beeckman's intention to publish, since he was indebted (and would probably be seen to be indebted) to Beeckman for the very conception of motion that lay at the heart of his physics.

2. CARTESIAN PHYSICS AND THE FORCE OF MOTION

Beeckman's "intellectual prize", according to Koyré, was his principle of the conservation of motion: *Once moved, things never come to rest unless they are impeded* (*JIB* I, 24; July 1613-April 1614). In the form he had proposed it to Descartes in 1618 it ran: "*what is once moved in a vacuum moves always*." (*JIB I*, 132). A fuller version penned in his *Journal* at around the same time ran:

> *Whatever is once moved moves always in the same way until it is impeded by something extrinsic*.[11] (*JIB* I, 256; Nov. 23–Dec. 26, 1618)

Here Beeckman's "decisive progress" consists in asserting that it is motion itself that is conserved, thereby emancipating the idea of motion from the necessity of an internal cause, as posited in the theory of impetus.[12] Koyré calls this a "prize" because he sees it as a close anticipation of the Newtonian principle of inertia, according to which no force is necessary for the continuation of motion in a straight line with the same speed. But he criticizes de Waard for overstating the case for this anticipation, in that Beeckman had explicitly interpreted "*in the same way*" as applying to circular as well as rectilinear motion, "without noticing," says Koyré, "that the conservation of circular motion and that of straight line motion are *strictly incompatible*" (*Galileo*, 118, n. 61). For not only had Beeckman first applied his principle to celestial and terrestrial circular motions, he had even given a formulation of it that shows he clearly did not see the incompatibility:

> *That which is once moved in a vacuum moves always*, whether in a straight line or in a circle, not only on its own center, as with the diurnal motion of the Earth, <but also around a center, as with> the annual motion of the Earth.[13]

What prompts Koyré to claim that Descartes initially "threw away" Beeckman's prize is Descartes' interpretation of this principle in terms of the conservation of *force*. For this implies, so Koyré maintains, a reversion to the impetus physics of the Aristotelians, according to which no motion occurs without a motive force. "He imagines that if the motion of the falling body accelerates this must be because it is attracted more strongly by the earth at the end of its motion than at the beginning, or to put it in his own terms, because the earth's attractive force produces a growing motive force in the stone." (*Galileo,* 83-84). Only later, as the fruit of a profound intellectual revolution in his thought, did Descartes come "to set out for us with unsurpassable clarity the new concept of motion, the foundation of the new science, and to determine its structure and ontological character" (89). On this new conception, motion is understood not as "the philosophers" understand it, but as the geometers do, when the motion of a point traces out a straight line or a circle. But "motions such as these," Koyré asserts, "in contrast to physical motions, have no speed and do not take place in time" (91). They are static. Motion is therefore a *state* and not a *process*, so that it does not need a cause for its continued existence any more than any other quality. This provides Descartes with the general foundation for his first Rule or Law of Nature in *Le Monde*, his "law of inertia":

> That each particular part of matter continues always to be in the same state so long as no collision with others compels it to change. That is to say, if it has a certain size, it will never become smaller unless others divide it; if it is round or square, it will never change this shape without others compelling it to; if it is at rest in any place, it will never leave it except if others drive it out; and if it once begins to move, it will continue always with the same force until others bring it to rest or slow it down. (AT XI, 38)

After articulating a second rule concerning the equality of quantity of motion gained and lost by two bodies in a collision, Descartes justifies both rules by an appeal to the immutability of God's action:

> These two Rules follow manifestly from the sole fact that God is immutable, and that, acting always in the same way, he always produces the same effect. For supposing he placed a certain quantity of motion in matter as a whole from the first instant he created it, we must admit that he always conserves the same quantity of motion in it, or else not believe that he acts always in the same way. (AT XI, 43)

All that needs to be added to yield Newton's inertial principle, says Koyré, is added by Descartes in his third Law of Nature in *Le Monde*, which specifies that since God conserves motion as it is in a given instant, and "only motion in a straight line ... may be wholly grasped in an instant," the determination of a body to motion that is conserved by God is necessarily in a straight line:

> This Rule is based on the same foundation as the other two, and depends only on the fact that God conserves each thing by a continuous action, and that consequently he conserves it not such as it may have been some time earlier, but precisely such as it is at the very instant that he conserves it. Thus it is that of all motions, it is only motion in a straight line that is entirely simple and whose whole nature can be comprised in an instant. For in order to conceive it, it suffices to think that a body is in the process of moving in a certain direction, which happens at each of the instants that can be determined during the time it is moving. (AT XI, 44-45)

In contrast, says Koyré, "the new idea of motion was far from clear to Beeckman" (*Galileo,* 120, n. 86). In fact, "Beeckman was so far from understanding his own ideas that he denied the continuity of acceleration that takes place during fall". Here Koyré alludes to the fact that Beeckman thought that all increases in motion are imparted by discrete impulses, and therefore occur in discrete increments, contrary to the continuity of acceleration in fall.

Thus Koyré's interpretation of the Cartesian theory of motion. Whatever its initial appeal, however, I do not believe it can be sustained, especially in the light of more recent scholarship. But before getting into the details of that, it is worth noting the implausibility of the scenario that Koyré depicts concerning the interaction of the two men. We are asked to believe that Beeckman, in arriving at his cherished principle of the conservation of motion, made decisive progress in abandoning impetus physics and purifying the concept of motion of the motive force that was its supposed underpinning; yet that “he did not understand the implications of his ‘principle’” (120, n. 86); so much so, that when Descartes reinterprets it in terms of conservation of force, he does not even demur. Descartes, for his part, despite this initiation into Beeckman’s mathematico-physics, interprets the increase in motion produced by the attraction of the earth in Beeckman’s scheme as an increase in the force of motion; and when he returns to the problem in 1629 at the behest of Mersenne —despite the fact that he has recently read Beeckman’s *Journal* and discussed the problem of fall with him without any evident misunderstanding— Descartes “reverts to the concept of *impetus*” so that “his interpretation of fall differs only very slightly from those which had been given by Benedetti and Scaliger” (87). Yet within a year or so (the period in which he wrote the first seven chapters of *Le Monde*), Descartes had arrived at a wholly modern conception of the principle of inertia, apparently at one stroke, as a result of his new method and epistemology.

Where Koyré’s interpretation comes decisively unstuck is in its claim that in *Le Monde* and afterwards Descartes conceives of motion as force-free. For he does not. As modern scholarship has shown, not only Descartes’ laws of collision, but also even his advances in optics, depend crucially on the concept of the force of a body’s motion. Indeed, this is evident even in the wording of Descartes’ “law of inertia” quoted above: “once [a part of matter] begins to move, it will always continue *with the same force* until others bring it to rest or slow it down.” (AT XI, 38) Granted body as extension contains no hint of force; but, as Guéroult has pointed out, this would apply only to body considered in the abstract. In order to exist, a thing must be created by God’s action, and the force or action by which God creates a body at an instant is in the body.[14]

Hence Koyré is mistaken in thinking that the basis of Descartes' new conception of motion lies in his geometrized notions of matter and motion. Each part of matter depends on God's force or action to sustain it in its given state, whether of motion or of rest, and this is manifested phenomenally as a force of continuing in motion or resisting motion, respectively. Created body, then, contains this force within it. Descartes is explicit about this in a letter to Mersenne of October 1640, commenting on a manuscript that had been sent him:

> He is right that it is a grave mistake to admit as a principle that no body moves itself. For from the sole fact that a body has begun to move, it is certain that it has within itself the force of continuing to move; likewise, from the sole fact that it is at rest in some place, it is certain that it has the force of continuing to stay there. But as for the principle of motion, which he imagines to be different in every body, this is wholly imaginary.[15]

This point of view is maintained by Descartes in his *Principia*:

> In this connection, though, one must pay careful attention to what it is that constitutes any body's force of acting on another, or of resisting the action of another. It consists solely in this: that each thing tends, so far as it can, to persist in the same state it is in, as laid down in our first law. Hence that which is attached to another thing has some force of resisting being detached; that which is detached, a force of remaining detached; that which is at rest has some force of persevering in its rest, and consequently a force of resisting all those things that could alter its rest; and that which is in motion has some force of persevering in its motion, i.e., in a motion of the same speed and in the same direction. (AT VIII.1, 66-67)

Descartes goes on in the *Principia* to deduce his seven Rules of Collision from estimates of this last force, which must depend "not only on the magnitude of the body it is in, and that of the surface by which this body is separated from the other, but also on the speed of the motion, and on the nature and contrariety of the way in which different bodies collide" (AT VIII.1, 67). Although he does not give any rules of collision in *Le Monde*, he claims that he could: "I could set out many further rules for determining in particular when, how, and by how much the motion of each body can be changed and increased or decreased by colliding with others" (AT XI, 47). Assuming that Descartes did indeed have some rules of collision when he wrote *Le Monde*, there seems little reason to doubt that, in whatever other respects they may have differed from the later rules in the *Principia*, they would have embodied a similar estimate of the force of motion in terms of the speed and magnitude of the body, among other factors. And as Schuster and Gaukroger have argued, this general conception of motion as being accompanied by a force of motion (with both an absolute quantity and a directional component) is a feature of Descartes' physics going back to his earliest work shortly after his collaboration with Beeckman.[16]

This is certainly not to say that Descartes made no advances in the understanding of motion after his collaboration with Beeckman. In particular, there is everything connected with his novel way of analyzing motion by investigating the diverse tendencies or actions operating at each instant. This is, I believe, an innovation of tremendous significance. Together with his insistence that it is only rectilinear motion "whose whole nature may be comprised in an instant", this paves the way not only for Descartes' own considerable advances in optics, but also for the mathematization of motion itself and its later connection with the calculus.

This advance over Beeckman can be seen clearly in Descartes' analysis of the conservation of circular motion. As we have seen, Beeckman understood his principle that "*Whatever is once moved moves always in the same way until it is impeded by something extrinsic*" (*JIB* I, 256) to entail that a body in circular motion about a center in a void would continue to move in a circle if nothing extrinsic acted upon it, thus explaining the motion of the planets, which move in a near void. In fact, he went so far as to claim that "since any least part of the circumference is curved, and curved in the same way as the whole circumference, there is no reason why the circular annual motion of the Earth should leave this curved line and proceed along a straight one" (*JIB* I, 253; 23rd Nov-26th December 1618). According to Descartes, on the other hand, such a body "always has a tendency to go in a straight line, and goes in a circle only under constraint" (*Le Monde*; AT XI, 44): if no extrinsic force were acting on a planet moving in a void, it would move off in a straight line tangent to the circle. For him this indicated that the planets clearly do not move in a void, a fact that might conceivably have contributed to his rejection of Beeckman's atomism. Once the void has been rejected, however, bodies never do undergo strictly inertial motion. According to Descartes' physics, motion in a straight line with no external forces acting is contrary to fact, since motion in a plenum is always constrained by the actions of surrounding bodies. In the case of a body constrained to move in a circle, as in Descartes' famous example of the motion of a stone in a sling, the God-given rectilinear motion makes it tend to move along the tangent, and thus away from the center; but this centrifugal tendency is balanced by the force of tension in the sling, resulting in an equilibrium. Conservation of circular motion is thereby explained in terms of a balance between forces normal to the motion, without recourse to Beeckman's "circular inertia".[17] Thus even if Descartes is not as close to a Newtonian conception of inertia as Koyré claimed on his behalf, his account of the conservation of (orbital) circular motion still represents a decisive advance over Beeckman's.[18]

Nevertheless, if Gaukroger and Schuster are correct, these advances concerning the directionality of motion and tendency to motion at an instant were probably made by Descartes in the early 1620s, as an immediate consequence of his own contributions to his initial collaboration with Beeckman.[19] So Descartes' understanding of motion did not undergo any "profound change" or "intellectual revolution" in 1630. And if we put to one side all such considerations about direction and tendency and concentrate only on motion itself—its nature and the force accompanying it—I submit that Descartes' understanding of this when he was writing *Le Monde* remained precisely as it had been earlier: a body in motion will continue to move with the same quantity of force (proportional to the magnitude of the body and its speed) until acted upon by extrinsic causes. But if this is so, I believe it obliges us to reconsider in detail the collaboration between Descartes and Beeckman of 1618-19.

3. Force and Motion in the problem of Fall

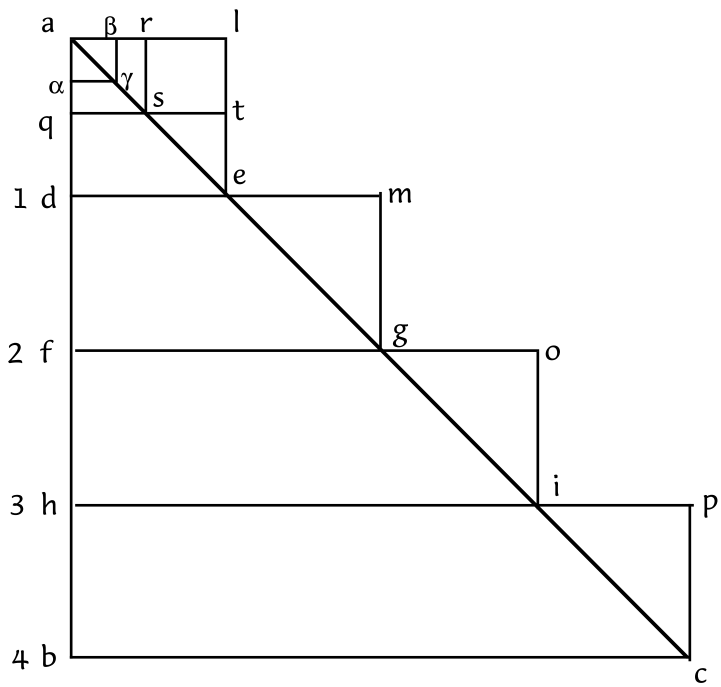
The challenge problem Beeckman set for his new friend "Mr. René du Peron" in late 1618 was to find out

> whether it is possible for someone to know how much space a thing would cover by falling for a single hour when it is known how much it would cover in two hours, according to my foundations, *viz.*, *what is once moved in a vacuum moves always*, and supposing that there is a vacuum between the earth and the falling stone. (*JIB*, I, 263; Nov 23rd–Dec. 26th 1618)

We do not know precisely what Beeckman might have told Descartes of his physics at this time. This challenge problem appears to be the first of the three that Beeckman communicated to Descartes in 1618, so their collaboration was in its earliest stages. Still, Beeckman's principle of the conservation of motion in a vacuum ("*what is once moved in a vacuum moves always*") is clearly contrary to standardly accepted Aristotelian principles, and whatever precedents for it Descartes might have encountered in his Jesuit education at La Flèche, it is reasonable to suppose that he would not have proceeded until he had understood it. How then does Descartes understand Beeckman's proposal? Here are some relevant passages from his solution, omitting for now the mathematical demonstration itself:

> In[20] the problem posed, in which it is imagined that at each time a new force is added by which the heavy body moves downwards, I say that this force is increased in the same way as are the transverse lines *de, fg, hi*, & an infinity of other transverse lines that can be imagined between them. To demonstrate this, I shall assume for the first minimum or point of motion, which is caused by the first attractive force of the earth that can be imagined, the square *alde*. For the second minimum of motion, we will have double this, namely *dmgf*: for the force which was in the first

minimum persists, & another new one accrues equal to this. Likewise in the third minimum of motion ...



... From which it clearly follows that if we imagine, for example, that a stone is pulled from *a* to *b* in a vacuum by the earth, by a force which always flows uniformly from it, with the prior force persisting, then the first motion at *a* is to the last at *b* as the point *a* is to the line *bc*; in fact the half *gb* will be traversed by the stone three times as fast as the other half *ag*, since it is pulled by the earth with a force three times as great: for the space *fgbc* is triple the space *afg*, as is easily proved; & one can treat the other parts of the line *ab* proportionately. (*JIB* IV, 49ff; AT X, 75ff)

I believe we can infer from this that Beeckman supplied Descartes with his formulation of the problem, which in the *Journal* runs as follows: “In the first moment, as much space is covered as can occur as a result of the traction of the Earth. In the second, with [the stone] persevering in this motion, a new motion of traction is added, in such a way that twice the space is traversed in the second moment.” (*JIB* I, 261; AT X, 58). Descartes interprets ‘moment’

(Latin: *momentum*) as a *minimum of motion*, i.e. in its root sense as a contraction of *movimentum*. As we shall see, this is in accordance with Beeckman's conception of motion as occurring in discrete increments, so that the "mathematically divisible space" covered is divided into "physical minima through which the attractive force moves the thing". A moment of time, correspondingly, is for Beeckman also a physical minimum, "namely, that in which the thing covers one physical minimum of space".[21] Now, in accordance with Beeckman's instructions, Descartes assumes that the force of motion accruing in the first moment persists, and in the second moment "another new one accrues equal to this". He then gives an elegant mathematical solution, which in certain respects, as I shall argue, goes beyond anything Beeckman had anticipated. But he seems not entirely sure that he has correctly interpreted what Beeckman meant by "a new motion of traction is added", so that, after giving his solution, he offers another interpretation of Beeckman's problem, one that is admittedly "more difficult":

> Let us imagine that the stone remains at the point *a*, the space between *a* and *b* being a vacuum; and now at the first time —for example, today at nine o'clock— let God create an attractive force on the stone; and let him create again and again at every single moment thereafter a new force equal to that which he created at the first moment; which, joined with the force created before, pulls the stone more and more strongly, since *what is once moved in a vacuum moves always*; and finally, let the stone, which was at *a*, arrive at *b* at ten o'clock today.
>
> If it is asked how much time it would take to cover the first half of the space, namely *ag*, and how much the remainder: I reply that the stone would descend through the line *ag* in a time of 7/8 of an hour, and through the space *gb*, on the other hand, in 1/8 of an hour... (AT X, 77)

Here Descartes interprets Beeckman's "a new motion of traction is added" as a stipulation that *a new force of traction is added with the prior force of traction remaining*, rather than simply its resultant force of motion remaining, as in the previous version. No doubt Beeckman soon set him straight that the first interpretation was the right one. But it is noteworthy that in this second version Descartes makes explicit what was left unsaid in the first version, namely that it is God who creates the attractive force on the stone in each moment.

So we see that Descartes interprets Beeckman as follows: after God has created a force of motion in the stone in the first moment, the motion persists in a vacuum with the same force from then on, in accordance with Beeckman's conservation principle, which Descartes explicitly quotes. Then at each subsequent moment a new force of motion is created in the stone equal to this.

But to this it may be objected on Koyré's behalf that (i) Beeckman does not talk of the *force* of the body's motion, but only of the *motion* being conserved; and (ii) in any case, this new conception does not seem to have impressed itself indelibly on Descartes' mind, since in his letter to Mersenne in 1629 he reverts to the concept of impetus.

The first of these objections rests on a misconception resulting from reading Beeckman's words from a post-Newtonian perspective. In stating his principle, it is true, Beeckman speaks only of the *motion* being conserved. But it does not at all follow from this that he thought that no force accompanies the motion. To the seventeenth century mind it was an empirical fact that a body in motion has a force of motion, as evidenced by its ability to set other bodies in motion when it collides with them, and its being necessary to apply a force to bring it to rest. The Scholastic theory of *impetus* is one interpretation of this force of motion. Although it existed in many varieties, the core conception is that *impetus* is a quality impressed upon the body which inheres in it, and is the cause or principle of its continuing motion.[22] Now by 1614 Beeckman has already explicitly disavowed this idea of impetus as an internal quality causing motion, as is evident from the following passage from his *Journal*, a passage Koyré no doubt had in mind in offering his interpretation of Beeckman as rejecting a force of motion altogether:[23]

> **A stone thrown in a vacuum does not come to rest:** Thus a stone thrown in a vacuum moves perpetually; but it is obstructed by the air which always

> runs up against it anew, and makes its motion diminish. Indeed, what the Philosophers say about a force being impressed upon the stone seems to be without reason; for who can conceive what this could be or how it could keep the stone in motion, or in what part of the stone it could be found? But it comes very easily to mind for someone who conceives that motion in a vacuum never comes to rest because no cause occurs to change the motion; for nothing changes unless there is some cause of change. Thus if you put one thing on top of another and move both of them together, and suddenly pull one of them back, the other will nonetheless keep moving.[24]

There is no doubt that this passage does indeed represent “decisive progress”, just as Koyré maintained, since Beeckman clearly rejects the Scholastic interpretation of *impetus* as a quality impressed on and remaining in the moving body as a cause of motion, or as a quality accruing to the moving body. This is explicit in the *Corollaria* to the *Theses* he published in 1618:

> A stone thrown from the hand continues to move not because of some force accruing to it [*ipsi accedentem*], nor for the sake of avoiding a vacuum, but because it is impossible for it not to persevere in the motion it had when it was in the hand. (*JIB,* IV, 44; see also *JIB,* I, 200-210).

But in rejecting impetus as a cause of motion Beeckman does not thereby reject the idea of a force of motion possessed by the moving body in the sense of a measure of its power to persist in its motion,[25] and thereby to set other bodies in motion, a power that is maintained, on his view, by God’s action. Natural philosophers of the time often used the word *impetus* to describe a force of persistence, with no commitment to the Scholastic theory of impetus as a *cause* that is used up in producing motion as an effect: indeed Koyré himself notes an instance of Galileo’s using the word in this way.[26] Beeckman also uses the term *impetus* for the power of a body to persist in motion, as is shown by this entry from his *Journal*, dated 25th–29th July 1619:

> **The same impetus sometimes moves a ship in a contrary direction to what it had before**: If a ship takes down its sail and is carried only by its prior impetus, it can be so guided by its rudder that, making a semicircle of

> its motion towards the same place it came from, it is moved along that line by which it had come, <and> returns by one and the same impetus. (*JIB* I, 330)

Granted, Beeckman's talk here of impetus "moving a ship" seems to imply its being a cause, and perhaps shows his indebtedness to earlier conceptions. But it should not mislead, especially in view of what he had already explicitly said about no internal cause being necessary for the continuance of motion. (We still talk in the same way of being "carried by our momentum", without imagining it as an internal cause of motion.) Beeckman, moreover, is quite explicit in giving the measure of this impetus or force of continuing in motion in terms of its *power to move other bodies in collision*, just as Descartes will do in his Third Law of Nature. It depends, like Descartes' quantity of motion, on quantity of matter (or "corporeity" [*corporeitas*] to use Beeckman's word), and on swiftness [*velocitas*]. For instance, in the case where a moving body hits one at rest, after which they move off together, Beeckman writes:

> And it is the same if some body at rest is knocked into by any body in motion. The one that was at rest will move with a motion in this manner: *If each body has the same corporeity, both will be moved twice as slowly as that which was moving first moved*. For since there are just as many parts in the body at rest as in the moving one, the moving body also applies an equal progress to it, that is, since the same impetus should be sustained by a body twice as large as before, it is necessary that it also proceed more slowly by the same amount.[27]

Obviously, this point is also highly germane to the second objection noted above, the contention that Descartes could not have understood Beeckman aright in 1618, since (Koyré asserts) in his letter to Mersenne of November 13th 1629 he "reverts to the concept of *impetus*, and his interpretation of fall differs only very slightly from those which had been given by Benedetti and Scaliger" (87). Now it is true that in this account of November 1629 Descartes writes of a weight being "impelled by its own heaviness", which "pushes it downwards, giving it at each moment a new force which makes it fall"; and that this sounds like previous accounts in which "gravity, an essential property of the body, produces a new impetus at each moment", with these successive increments of impetus themselves acting as "accidental gravity" causing successive increments of motion downwards. Indeed, Descartes writes of the weight's subsequently "retaining all the impetus" it had in the first half of its fall. Koyré claims that this "amounts to a transposition into *impetus* terminology of the conception which had been developed in terms of attraction" (87), and that Descartes is here deliberately abandoning the concept of action at a distance, an idea that would have had no attraction for him (if this expression may be pardoned!)[28]

To this it must be responded, first, that by his mere use of the word, Descartes has no more reverted to the theory of impetus than had Beeckman or Galileo. He does not believe that gravity is an essential property of a heavy body, and nowhere asserts or implies that impetus is an internal quality used up in producing the motion, or a cause expended in producing motion as an effect. Indeed, he begins his account in the November letter to Mersenne with a clear statement of Beeckman's principle of the conservation of motion, which entails that he must also be interpreting *impetus* à la Beeckman as the force of continuing in motion, but not as an internal cause:

> First I suppose that the motion that is once impressed on some body remains in it perpetually unless it is taken away by some other cause; in other words, that what has once begun to move in a vacuum moves always and with an equal speed.[29]

The cause of the motion, that is, is God's action in creating the force of heaviness [*gravitas*] at each moment. Assuming that God continues to act in the same way, no cause is required for the continuance of this same motion of the body, nor for its continuing with the same force of motion or impetus. That is, assuming the constancy of the primary cause, no secondary cause is required for a continuing motion. But since the heaviness of a falling body continues to act as a (secondary) cause, it therefore "acquires an additional impetus on account of [this] heaviness which impels it afresh at each moment" (*ibid.*).

This idea of impetus "impelling" the falling body afresh at each moment appears to privilege impetus as a cause of motion intermediate between the force of gravity or heaviness and the motion itself.[30] But the language is no different from Beeckman's own in the passage quoted above about impetus "moving a ship". If we allow some metaphorical license and concentrate on the fact that in each case the dependence of motion on an internal cause is precluded by the appeal to Beeckman's conservation principle, the difference seems more one of style than substance.

Second, it seems clear that Descartes did not take his description of the account of fall in terms of the acquiring of impetus to be in contradiction to Beeckman's account. Had he done so, one would have expected him to draw attention to this difference in his letters to Mersenne after October 8th, 1629, when he cut himself off from his former friend. This would have been particularly so in his letter of 18th December, since in it he was responding to "something concerning the speed of motion which you say that Sr. Becman sent you". But instead of confronting this directly, he chooses to reply first to Mersenne's misunderstanding of his own previous account, in which Mersenne had imputed to him the view that "the speed is impressed as 1 in the first moment, and 2 in the second moment, etc." Descartes corrects him, explaining his own view as follows:

> [T]hat is not how I understand the matter. Rather, the speed is impressed by the heaviness [*gravitate*] as 1 at the first moment, and again by the same heaviness as 1 at the second moment, etc. ... This is sufficiently proved, I thought, by the fact that the heaviness perpetually accompanies the body it is in, and that heaviness cannot accompany a body without incessantly pushing it downwards. Now if we suppose, for example, that a leaden mass is falling downwards by the force of its own heaviness and that, after it has begun to fall through the first moment, God removes all the heaviness from the lead, so that after this the leaden mass is no heavier

> than if it were air or a feather, this mass will nonetheless continue to fall, at least in a vacuum, since it has begun to move, and no reason can be given why it should stop; but its speed will not increase. (It should be remembered that we are supposing that what is once moved in a vacuum moves always,[31] as I shall try to demonstrate in my treatise.) But if after some time God should restore the heaviness to the lead for a moment of time only, and after this has elapsed take it away again, wouldn't the force of heaviness impel the lead in the second moment just as much as it did in the first moment? And wouldn't the speed of motion be twice as great? The same should be said of the remaining moments.[32]

This is surprisingly similar to the account that Beeckman himself had sent Mersenne on October 1st of the same year, using the same idea of taking away the gravity of the falling body after the first moment in order to bring home the principle of conservation of motion. There Beeckman had written:

> I am surprised that you do not believe that a falling stone, before it reaches the point of equality, increases in speed (*celeritate*) at every single moment. For if the attracting (*trahens*) force does indeed attract at the next moment, but not at the second and third moments, no one would deny that the same thing would happen to this stone as a result of this force as would happen in violent motion, that is, it would continue to move for as long as the stone is projected. But now the Earth will pull at the <second> and <third> moments.[33] Thus at the first <moment> it pulled the stone at rest, but at the remaining <moments> it pulls the stone already existing in motion towards the Earth with forces equal to the first. Therefore at the second moment it is moved partly by the motion remaining from the first moment, partly also by the new motion of the second moment, and, unless the air is an impediment, the motion of the falling stone will increase as far as the center of the Earth, where what you say about its being penetrated will occur.[34]

Now Descartes had not read this letter. Nevertheless, he not only tells Mersenne that "what he has sent you is false, to wit that there would be a place that a falling weight would reach after which it has an always equal speed", but considers himself sufficiently familiar with Beeckman's account to be able to "explain to you what he meant to say, for we have discussed this together in the past." What is at stake, then, is just how his own views agree with and differ from Beeckman's. Thus at this point, if he understood his account to be in opposition to the "attractionist" account given by Beeckman, this would have been the place for him to say so. Instead he says:

> He supposes, as I do, that what has once begun to move continues to move of its own accord, unless it is impeded by some external force, and therefore in a vacuum it moves always, but in air it is gradually impeded by the resistance of the air. He supposes further that the force of heaviness in a body, existing at every single moment imaginable, impels the body anew so that it descends, and therefore in a vacuum the speed of motion is always increased in the proportion that I reported above, and which I investigated when he proposed it eleven[35] years ago, and which I still have among my notebooks annotated from that time. (AT I, 91; *CMM* II, 339)

Setting aside the implicit claim that the principle of conservation was something they shared rather than original with Beeckman,[36] what is notable about this passage is that Descartes does not repudiate Beeckman's physics of fall *in vacuo*, but endorses it, and reports it in the same terms as his own.

Third, lest it be thought that Descartes in December 1629 is either misremembering Beeckman's idea of the "traction of the Earth", or simply carelessly re-expressing it in his own terms, it must be replied that this seems unlikely given the close familiarity he shows with Beeckman's views on motion in air in the continuation of this passage. The accuracy of this account seems explicable only by a close familiarity with Beeckman's views as expressed in his *Journal*.[37] For what Beeckman proposes there concerning motion in a plenum (both before he meets Descartes, again at their reunion in Dordrecht in 1628, and still in 1629) is a hydrostatic model. On this account, when a stone falls in the air a point of equilibrium will be reached "where the impediment of the air is as strong as the motion". The reason for this is that "the faster a body descends, the more the air resists its motion", a thesis original with Beeckman, as Descartes admits, and about which he reports that he was "doubtful earlier" [i.e. in 1618], "but now that I have carefully examined the matter [i.e. in December 1629] I can see that it is true". Descartes continues:

> From this he draws the following conclusion. The force that creates speed always increases uniformly (that is, by one unit at each moment), while the air resistance always impedes it in a non-uniform way (that is, less than a unit at the first moment and a little more at the second moment, and so on). Necessarily therefore, he says, there comes a point where this resistance is equal to the thrust which is due to the heaviness, when it reduces the speed at the same rate as the force of heaviness increases it. At the moment this happens, he says, it is certain that the weight does not fall more quickly than it did at the immediately preceding moment, and at the subsequent moments the speed will neither increase nor diminish, because after that the air resistance remains uniform ... and the force of heaviness always pushes it in a uniform way. (AT I, 91; *CMM* II, 339-340)

This is an accurate and faithful representation of Beeckman's views, right down to the representation of motion in terms of discrete moments during which the increments of motion are understood to occur, as can be seen by comparing the entry in Beeckman's *Journal* for December 1618.

But what of motion in a vacuum, and the "traction of the Earth" that Beeckman assumes as the cause of heaviness? Isn't Koyré right in seeing this as something unacceptable to Descartes?

Here I think two questions need to be distinguished: that of the cause of gravity, and that of the cause of increments in the force of motion. With respect to the first, Descartes, it is true, does not appear to subscribe to Beeckman's hypothesis that the magnetic traction of the Earth is the cause of gravity; but this is not because Descartes is a consistent mechanist, while Beeckman believes in action at a distance. Such a way of conceiving the matter again errs in presupposing a post-Newtonian perspective, where an attractive force is interpreted as a *cause*, and one that precludes a mechanical explanation. No such theory was entertained by Beeckman, as is borne out by an examination of the *Journal* and his correspondence with Mersenne, where he can be seen seeking an explanation of the force of attraction in terms of discrete impacts of “corporeal spirits” or magnetic atoms.[38] This in itself would not have been unacceptable to Descartes; we have already noted that his mature micro-mechanical explanation of magnetic attraction in terms of screw-shaped particles was taken by him from Beeckman. Presumably he simply did not agree that gravity was caused by magnetism.

But of course this minor disagreement does not prevent Descartes from taking over the crucial feature of Beeckman’s account that gravity—no longer regarded as an intrinsic property of bodies—stands in need of an external, and indeed micro-mechanical, cause. Moreover—and this is the key point here—once the force of gravity is conceived in Beeckman’s terms as producing discrete increments of motion in discrete moments, each being conserved, it is not necessary to have an account of the cause of gravity in order to treat the problem of fall. This, I believe, is quite sufficient to explain why Descartes does not dwell on this aspect of Beeckman’s solution in his discussion with Mersenne: all that is necessary for Beeckman’s account to go through is the premise that the force of gravity—whatever its cause—acts by producing discrete increments of motion in discrete moments, which are each conserved in the subsequent moments.

In favour of this interpretation, in 1636 Marin Mersenne also presents Beeckman’s account of fall in terms almost identical to Descartes’, despite the fact that in his correspondence with Beeckman only a few years earlier they had discussed the latter’s hypothesis of magnetic traction:[39]

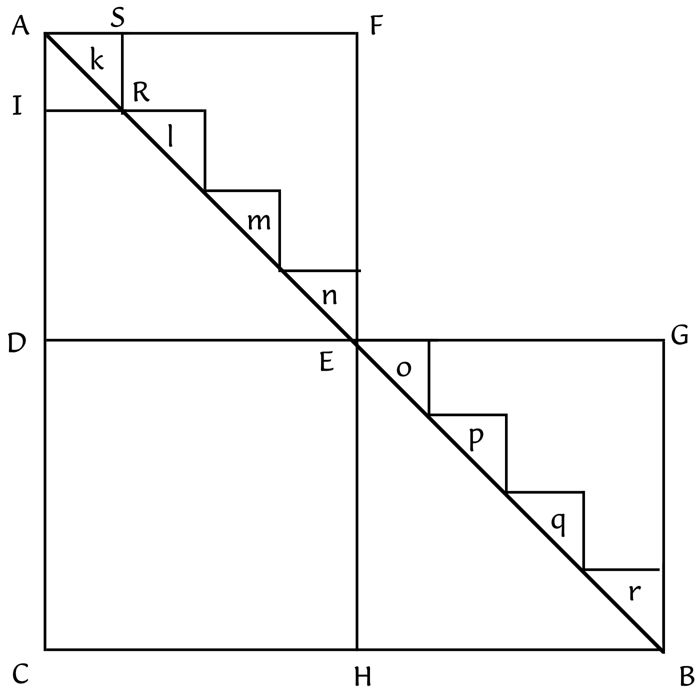
> [Beeckman] imagines that the weight, perpetually pressing the air, always increases its speed in such a way that, if after the first moment at which the stone is moving God himself took away its heaviness, it would still descend by the force of motion that is impressed at the first moment, and that if it were in the void, it would go always with equal speed; but because the weight always accompanies the first movement, it would accrue its

> speed by a degree at each moment, from which it follows that the stone makes no more headway in the first three moments than in the fourth.[40]

Thus Mersenne, like Descartes, reports Beeckman's explanation of the conservation of the force of motion without mentioning the cause of gravity. And, as had Descartes in his 1629 letter to Mersenne ("he supposes, as I do"), he avails himself of Beeckman's own example of the body's continuing to move with equal speed in moments subsequent to the first in which God had hypothetically removed its heaviness.

4. Discontinuity and Motion

Now all of this is not to say that Beeckman and Descartes agreed about the problem of fall, despite their collaboration and their sharing the results of that collaboration. To see this it is necessary to examine not only Descartes' solution to Beeckman's challenge problem in 1618 and Beeckman's very neat re-expression of it in his own words, but also the comments Beeckman makes immediately afterwards in reaction to it. The latter make it clear that, contrary to what Koyré and others have said, Beeckman himself was quite able to solve this problem in the terms in which he had given it. If the increments in motion are discrete (the result of individual impacts or tugs), occurring at the beginning of each successive equal moment or physical indivisible of time, then the speeds will be as 1, 2, 3, 4, etc in the successive moments; and given the equality of moments, the distances will therefore also be as 1, 2, 3, 4, etc in the successive moments, i.e. in arithmetical progression. (This is the same result that Leonardo had obtained earlier.[41])



What Beeckman receives from Descartes, however, is something unexpected: his new friend assumes that it is wrong to attribute a breadth to the minima, and claims that the moments of the motion “ought to be imagined as indivisible & as containing no parts” (*JIB* IV, 50). In Beeckman’s own words in his recounting of Descartes’ solution, “the quantity of these increments [of space] will be null when the quantity of the moment is set as null. But such is the moment of space through which the thing falls.” (*JIB* I, 262). Consequently Descartes effectively takes the limit of the total area as the moments —and therefore the protruding triangular spaces (*klmnopqr* in Beeckman’s diagram given above)— increase in number to infinity and as their breadth or quantity goes to zero. Moreover, contrary to the claims of Stillman Drake, this limit is taken correctly,[42] as Beeckman explains, by showing that “since these equal increments [i.e. the protruding triangles which represent the excess of the area calculated as a sum of discrete squares over the area of the triangles] always become smaller in the same ratio as the moments of space are smaller, it follows that the quantity of these increments will be null when the quantity of the moment is set as null". On this assumption that the quantity of the moment is null in the limit, it therefore follows that "the space through which the thing falls in one hour is related to the space through which it falls in two hours as the triangle ADE is to the triangle ACB." (262)

Beeckman understands the mathematics of Descartes' solution. However, he is not convinced that it is correct to suppose that the minimum of space has no quantity; rather, as we have seen, he endorses a discretist conception, where space is divided by the motion into finite physical minima. Moreover, if the minimum does have quantity, the problem cannot be solved from one case alone, since two cases are required to work out the size of the minimum of space. This is the solution he probably expected from Descartes:

> If, however, the moment or minimum of space has some quantity, there will be an arithmetic progression. Nor could it be known from one case how much [the falling stone] will cover in each hour; but there need to be two cases, in order for us to know from this the quantity of the first moment. Or so I had supposed; but since the supposition of an indivisible moment is more acceptable, I will not explain this in greater detail. (263)

Nevertheless, Beeckman observes, Descartes' result "that the space in the case of one hour is to the space in the case of two hours as *ade* to *acb*" can be obtained as a kind of limiting case

> when we consider that in an arithmetic progression the ratio of all the numbers comprised by half the terms is to the numbers of all the terms always [somewhat less than][43] 1 to 4, even if the proportion is perpetually increased. Thus the progression of two terms, which is 1, 2 is as 1 to 3. Thus 1, 2, 3, 4, 5, 6, 7, 8 is as 10 to 36. Thus the latter eight terms to 16 are as 36 to 136, which is not yet as 1 to 4. If therefore the fall of the stone occurs through distinct intervals, with the Earth pulling by means of corporeal spirits, these intervals or moments will be so small that their arithmetic proportion, on account of the multiplicity of particles, will not be

> sensibly less than 1 to 4. Therefore the aforesaid triangular demonstration should be retained. (263)

That is, the supposition of an indivisible moment is more acceptable precisely because the force of gravity is supposed to act mechanically, by discrete tugs or jerks. Descartes' continuist solution can nevertheless be accepted because the difference between this idealized case and the one that must obtain physically will be below any determinable sensible threshold. Any lingering doubt that this represents a real ontological difference between the two, even if it is empirically undeterminable, is removed by Beeckman a paragraph later:

> Indeed this triangular proportion was acceptable to us, not because there will not really be some physical minimum of mathematically divisible space, through which physical minima the attractive force moves the thing (for this force is not really continuous, but discrete, and, as they say in Flemish, "sy trect met cleyne hurtkens [it pulls by little jerks]," and therefore consists in the aforesaid increments, in an arithmetic progression); but it is acceptable, I say, because this <minimum> is so small and <in>sensible that, because of the multitude of terms in the progression, the proportion of numbers does not sensibly differ from the continuous triangular proportion. (264)

This, of course, sets in a different light Koyré's criticism that "Beeckman was so far from understanding his own ideas that he denied the continuity of acceleration that takes place during fall." He denied it precisely because it was inconsistent with his mechanistic ontology of motion and force, and he did so without in any way violating the continuity of motion implicit in his conservation principle. Here I believe Koyré was misled by de Waard's suggestion that Beeckman subscribed to the Arriagan theory of motion, according to which a slower motion is one interrupted by a greater number of rests (or perhaps rests of greater duration). I can find no evidence of this in Beeckman's work.[44] Motion itself, even though it changes in discrete increments, is continuous within the intervals between the "*horten*" or "*cleyne hurtkens*", consistently with the fact that in a vacuum it would continue unabated: "when [the attractive force is removed], the thing continues to move just as it would move having once been moved in a vacuum". The jerks or tugs of the corporeal spirits simply add incremental degrees of speed to the motion, which is uniform within each interval. This is why Beeckman insists that a thing falling in air will eventually come to a point of equality or constant speed, for there will occur a discrete first moment at which the quantity of air obstructing it in that moment will have the same "corporeity" (*corporeitas*) as the thing itself:

> if the parallelepiped which is described at such a moment contains as much corporeity as the thing itself contains, then the attractive force of the Earth will not be able to be add anything to the motion of the thing, because the heaviness [*gravitas*] of the body into which it is going, that is, the air, is equal to the heaviness ([*gravitati*] of the thing; for something existing in something equally heavy, such as water in water, will not move downwards. (264)

Descartes' disdain for Beeckman's discretist mathematics can be seen in his criticisms of this argument in his letter to Mersenne of December 1629. For having accurately reported the above argument, and allowed that it "is highly plausible", he writes scathingly that "those who are ignorant of arithmetic might be convinced by it; but one needs only to be able to count to see that it is unsound." (*JIB* IV, 172) For this he uses the idea of a continuous proportion between the speed and the resistance of the air to argue that an equality between force of motion and resistance can never be achieved, although it could perhaps be approached arbitrarily closely. Thus if a body falling in air that would have had a speed of 1 in the first moment has its speed reduced to $^1/_2$ by air resistance, then in the second moment a speed that would have been $^3/_2$ will be reduced proportionately by the same factor of $^1/_2$ to $^3/_4$; its speed in subsequent moments will then be reduced by the air resistance to "$^7/_8$, $^{15}/_{16}$, $^{31}/_{32}$, $^{63}/_{64}$, $^{127}/_{128}$, $^{255}/_{256}$ and so on to infinity... Thus the reduction in speed due to air resistance is never as great as the increase in speed due to heaviness, which is one unit at every moment". (172)

With this argument, Descartes has neatly turned the tables on Beeckman's criticism of his law of fall in a vacuum: for a body falling in air, it is Beeckman's solution that is a mathematical abstraction, but one whose difference from what would be observed experimentally is practically insensible.

5. BEECKMAN'S CONSERVATION PHYSICS

Thus, to encapsulate the discussion so far: notwithstanding the difference between Descartes and Beeckman over whether acceleration is discrete or continuous, there appears to be next to no difference of opinion between the two concerning the force of a body's motion. In fact, we may offer the following as a summary of the principles concerning motion that were first articulated by Beeckman, and subsequently taken up by Descartes:

1. ***No cause is required for continuance of motion in the same way.*** "In a vacuum ... no cause occurs to change the motion; for nothing changes unless there is some cause of change" (Beeckman, *JIB*, I, 1613-1614; 24). "The motion impressed on a body at one time remains in it for all time unless it is taken away by some other cause" (Descartes to Mersenne, 1629); "The philosophers have excluded motion from the rule—which is just the thing I most definitely wish to include in it" (Descartes, *Le Monde*, AT XI, 38).
2. ***All change of motion is through collisions.*** Implicit throughout both men's work; e.g. Beeckman's "*How motion in a vacuum is impeded by collisions*," etc. (Beeckman, *JIB* I, 265-266; 1618); and Descartes' rendering his claim in the Latin version of the *Principia*, II §37, that no state of a simple and undivided thing "is ever changed except through external causes" as "… except through collision with others" in the French version.
3. Therefore ***no motion will cease except as a result of collisions***. "*No thing, once moved, ever comes to rest, except because of an external impediment*" (Beeckman, *JIB* I, 24, July 1613-April 1614); "*whatever is once moved, moves always in the same way until it is impeded by something extrinsic*" (Beeckman, *JIB* I, 256; 1618). "Each particular part of matter always continues in the same state so long as collision with others does

not compel it to change that state; … if it once begins to move, it will always continue with the same force until others bring it to rest or slow it down.” (Descartes, *Le Monde*, AT XI, 38); “what is once in motion continues to move, until it is slowed down by bodies in its way” (Descartes, *Principia*, AT VIII.1, 63).

4. In particular, ***anything moved in a vacuum will always continue to move with the same speed.*** “*What is once moved in a vacuum moves always*” (Beeckman to Descartes, December 1618); “what is once moved in a vacuum moves always” (Descartes to Mersenne, December 1629). “According to this very often heard [principle] of mine, *what is moved in a vacuum always moves with the same speed of motion by which it began to move*” (Beeckman, *JIB* III, 185; 5th Jan.–10th Feb. 1631); “what has once begun to move in a vacuum moves always and with an equal speed” (Descartes to Mersenne, December 1629; *JIB* IV, 166); “that which is in motion has some force of persevering in its motion, i.e. of continuing to move with the same speed and in the same direction” (Descartes, *Principia*, AT VIII.1, 66-67).
5. Therefore ***the Scholastics are wrong to think motion would cease but for the continued action of a force.*** “What the Philosophers say about a force being impressed upon the stone seems to be without reason; for who can conceive what this could be or how it keeps the stone in motion, or in what part of the stone it could be found? But it comes very easily to mind for someone who conceives that motion in a vacuum never comes to rest, because no cause that could change the motion occurs” (Beeckman, *JIB* I, 24-25, 1613-1614). “The motion of which the Schoolmen speak has a very strange nature; for ... it has no other end or goal than rest, and, contrary to all the laws of nature, it strives of its own accord to destroy itself” (Descartes, *Le Monde*, AT XI, 40).
6. On the contrary, ***motion, having been created by God, is conserved by him to eternity***. “Motion, once it has been created by God, is conserved to eternity no less than corporeity” (Beeckman, 1620);[45] “God imparted various

motions to the parts of matter when he first created them, and he now conserves all this matter in the same way, and by the same process that he originally created it" (Descartes, *Principia II*, 62).

7. Accordingly, ***since impetus*** or force of a body's motion ***is not an internal cause of motion in a body,*** but rather a concomitant of the motion, ***it must be produced in it by the same external cause; namely, God.***
8. Therefore ***the overall quantity of this motion (i.e. the quantity of the force of motion) is conserved by God.*** "*God is the primary cause of motion; and he always conserves the same quantity of motion in the universe*" (Descartes, *Principia*, AT VIII.1, 61). The fact that quantity of motion is measured by "corporeity" times speed is implicit in Beeckman's rules of collision (*JIB* I, 266; 26th Dec, 1618); for instance, when a body collides with one at rest and then carries it off, the following rule applies: "*as both bodies together are to the one moving first, so is the speed of the one moving first to the speed of both together.*" As Beeckman says in 1629, "corporeity and motion are reciprocal to one another" (*JIB* III, 133-4; *CMM* II, 256).
9. Finally, **regarding gravity and the problem of fall**: Since there is no natural motion downwards, since only change of motion needs a new cause, and since all change of motion is by collisions of particles, ***increments in the speed of this motion must be produced by discrete impacts of corpuscles***.

What this encapsulates is a remarkably coherent ontology of motion, subscribed to by both Descartes and Beeckman. Moreover, whatever precedents one may rightly find for it in, for example, the theory of impetus of Buridan, the originality of the overall conception is beyond doubt.[46] But there also seems to be little doubt that, however much Descartes' enlightened education at La Flèche may have prepared him to accept such an anti-Aristotelian conception,[47] the account, from its breadth of scope down to the detailed enunciation of principles, is original with Beeckman.

Having observed the coherency of this ontology, I should say something in reply to the criticisms about the lack of a "system" here. Beeckman, it is true, never ordered his thoughts to give a clear and systematic presentation of his ideas, even taking some pride in the spontaneity of his meditations.[48] (John Schuster suggests that Descartes' instinctive preference for "method, order and style" was perhaps fortified by impatience with Beeckman's unsystematic approach.[49]) It is true also that Beeckman's mechanistic and empirical outlook was capable of tolerating numerous incompatible hypotheses that were consistent with his atomistic foundation; although this is no less true of Descartes' plenistic corpuscularianism. Nevertheless, even if the presentation of his ideas was not systematic, it does not follow that Beeckman was philosophically naïve, or that there was no coherent foundation to his natural philosophy.

Let me give an example to illustrate this point. Regarding the principle that "what is once moved in a vacuum moves always", Descartes had told Mersenne in 1629 that he would try to prove it in his treatise.[50] In his *Monde* it is derived from the more general principle of persistence quoted above that "each particular part of matter always continues in the same state so long as no collision with others compels it to change" (AT XI, 38). Likewise in the *Principia* it is cited as just one example of that general principle of persistence, along with a square thing's remaining square, and a body's remaining at rest: "If it is at rest we do not believe that it will ever begin to move unless it is impelled to motion by another cause. Nor, if it is moving, is there any reason for thinking that it would ever lose this motion of its own accord and without anything else obstructing it" (AT VIII.1, 62). But this does not differ significantly in either content or degree of rigor from Beeckman's own justification of what he calls his "theorem":

> For why should that which is once moved in a vacuum ever come to rest? And this is seen to be just as necessary as the principle that if something is once at rest, it should always remain at rest, as long as it is not moved by another thing. Even if, then, there is something in motion which we do not understand, and which is nevertheless given, it seems no less absurd

to <assert> that a body could come to rest by itself than to assert that bodies could vanish into nothing.[51]

Indeed, to be more provocative, there are certain respects in which Beeckman's atomist ontology is more coherent and systematic than Descartes'. There are, for instance, features of the above foundation of motion that are perfectly explicable on an atomist outlook, but which are either unmotivated or problematic within Cartesianism. Two examples will serve: "corporeity" or "magnitude" as a factor in the force of motion; and the idea of conservation of quantity of motion itself.

Beeckman's concept of the *corporeitas* of a body, which is a measure of the quantity of matter, and is proportional to weight, is a clear ancestor of the modern mass concept. This concept arises naturally within atomism as a quantity of atoms: for with two bodies of the same volume, that body will have more corporeity which has more atomic matter, or less void (assuming, naturally enough, that all atoms have the same degree of corporeity). In Descartes' philosophy, however, the notions of 'bulk' (*moles*) and density are problematic, since every equal volume has the same quantity of matter.

Second, as Bloch has explained, the idea of conservation of motion has its origins in Epicureanism, according to which every atom possesses a certain innate activity or *energeia*; such that, even though individual atoms come to a standstill for a time, the total *energeia* in the universe is a constant.[52] It is somewhat problematic how this is supposed to work: if two atoms—which necessarily have the same (maximum) speed—collide head-on, and one of them is stationary for a while before restarting, then motion is not conserved on the level of individual atomic collisions, though it is still supposed to be conserved overall. Beeckman recognized this as a problem, and finessed it eventually by dealing only with conservation at the level of concretions of atoms. On this conception a macroscopic body will have a quantity of motion proportional to its corporeity and to its velocity; and if it disintegrates into parts moving with the same speed, the aggregate of these parts will have the same corporeity, and therefore the same force of motion, as the original body: they will incite another body of the same size to move with the same speed as would the original body. Otherwise, as Beeckman writes in his *Journal* in 1629, if corporeity is not taken into account, each part of a disintegrated body will have "the same force of moving and resisting another as the whole" (*JIB* III, 129, 13th-30th Sept. 1629). Therefore, he concludes,

> whatever moves in a vacuum and collides with another thing at rest having the same weight (that is, having the same corporeal quantity, whether it occupies more or less place; for this makes no difference in a vacuum), will continue to move together with the thing that was at rest with half the speed with which it was moving by itself before; for it does not seem possible that it could carry something off with itself except by imparting to it as much of its own motion as is the proportion of the corporeity of the body at rest to the corporeity of the one moving. Therefore one atom will move even the whole Earth, but with a speed diminished by as many times as the corporeity of the atom goes into the corporeity of the terrestrial orb. (129)

The same general scheme can also provide an atomist foundation for the persistence of motion of individual macroscopic bodies. Each individual atom moves "inertially" with the maximum speed, while it is in motion. Taking into account all the individual collisions, it is reasonable to infer that there will in general be not only a fixed proportion of the atoms in an isolated body that are in motion at any given time, but also a certain net motion (a "drift velocity") in a certain direction, necessarily substantially less than the atomic speed. In a vacuum this motion will then be conserved, because no atomic collisions will be adding to or subtracting from it.

Things are not nearly so straightforward in Cartesian physics. In the first place, the law of persistence ("inertia") is not well motivated. In a plenum, a body simply cannot continue in the same state of motion "until it is slowed down by bodies in its way" (Descartes, *Principia* II, §37; AT VIII.1, 63), since there are always bodies in its way.[53] Moreover, if there is no motion in an instant, it is hard to see how there can be a quantity of motion or a state of motion in an instant either. As Daniel Garber has noted, this explains why Descartes formulates his second law in terms of tendencies to move rather than motions.[54] But by the same token, it means the first law can never apply in Descartes' *World*. Even the statement of the law in the *Principia* seems to betray its atomist origins:

> Each and every thing, insofar as it is simple and undivided, always remains, so far as it can, in the same state, nor is it ever changed except through external causes (*French version:* through collision with others). (AT VIII.1, 62)

This talk of simple and undivided bodies is clear on an atomist picture, but problematic on the Cartesian view. Since according to Descartes the division of bodies is effected by differing motions, and bodies are always to some extent colliding with others, it appears they cannot remain simple and undivided, let alone maintain the same state of motion.

Second, there are difficulties with Descartes' attempt to derive the principle of the conservation of quantity of motion from the immutability of God's creative action. As we saw in section 2 above, the conservation of the quantity of motion of the world is supposed to follow from the constancy of God's action in conserving matter in general; and the conservation of the quantity of motion in collisions is supposed to follow from the conservation of the total quantity of motion in the world. The first inference is already problematic insofar as Descartes' measure of quantity of motion includes the magnitude (corporeity) of the bodies involved, and the latter notion has no natural foundation in a plenistic physics. But even setting that aside, there is the difficulty observed by Daniel Garber that this principle of the conservation of the same quantity of motion in the world as a whole "says nothing about how motion is to be distributed among individuals in the world, whether it is to persist in individual bodies, or whether it is to redistribute itself promiscuously and arbitrarily from body to body" (*DMP* 215, 206). Beeckman, as we have seen, provided an argument in his *Journal* as to why quantity or force of motion should be preserved in individual collisions; Descartes appears to assume that the only way for it to be conserved in the whole world is for it to be preserved in individual collisions, and unconvincingly attempts to ground the former in the constancy of God's action.[55]

Now, as I explained above, I am not trying to deny the original and very important contributions that Descartes himself made to the theory of motion, especially in his formulation of the concepts of the tendency to motion at an instant, and of the determination of a body to move in a certain direction. The former especially is a seminal advance, crucial to the development of rational mechanics, and no hint of it can be found in Beeckman. Nevertheless it is worth noting that it is precisely in those areas where Descartes makes his clearest advances over Beeckman (viewed in hindsight) that he has most problems with consistency. Beeckman's ontology of force, for example, is entirely consistent with his mechanism. Since all changes of motions are the result of collisions, all increments or decrements of motion are discrete. Descartes' superior handling of the mathematics of the problem of falling bodies allows him to be supercilious about Beeckman's discretist efforts. Nevertheless, his prized solution of the problem of fall using geometric algebra is not something he is able to include in his published physics, since its assumption of a continuously acting force is at variance with the mechanist ontology of forces resulting from discrete and discontinuous impacts that he shares with Beeckman.

Again, Beeckman's discrete moments, although very small, are of finite quantity. It is therefore consistent to regard a body as in a state of motion in any one of these moments, and as possessing a quantity of motion. Descartes' method of shrinking these moments down to zero quantity and considering the state of motion at the very beginning of each moment will, in the hands of Newton and Leibniz, usher in the calculus and thus the whole modern age of mechanics. But in his own hands it is not yet coherent: without the concept of instantaneous velocity, there is no quantity of motion at an instant; and there is no justification for his claim that tendencies to move will follow the same laws as motion itself. Even Descartes' original contribution of identifying the directionality of motion at each instant as a crucial component in the analysis of motion is problematic. According to Descartes, a body's determination to move in a given direction is an independent property of the motion, just like the quantity of its motion. But unlike the latter, it has no quantitative measure: in arriving at his rules of impact, Descartes uses a model derived from statics to decide which body's determination prevails.[56] Again from the vantage of hindsight, this contrasts unfavorably with Beeckman's (albeit incomplete) analysis of collisions, which is correct for the simple cases he considers.

In conclusion, a great deal of what is normally ascribed to the genius of Descartes should instead be ascribed to his erstwhile friend and mentor. Without denying Descartes' undoubted contributions, he did not have to "set out for us with unsurpassable clarity the new concept of motion, the foundation of the new science, and to determine its structure and ontological character" (Koyré, *Galileo*, 89), since this task had largely been accomplished by Beeckman, from whom he had inherited the new concept of motion. Similarly, when Peter McLaughlin writes of Descartes' concept of force in a recent article that the kind of force that is "definitive of the [Cartesian] system of matter" is "the causal action (God or whatever) that is conserved in the world system", and that "incorporating this [notion of force] into the conceptualization of modern physics is Descartes' major contribution to modern physics" (*DNP*, 83-84), he should be corrected: this is *Beeckman's* major contribution to modern physics. In fact, I believe one can go further: to the extent that Descartes learnt the above nine principles of motion from Beeckman, acknowledged that he did, and then subsequently and very publicly denied he had learned anything from Beeckman, used his influence to suppress knowledge of his debt to his former mentor, and took sole credit for their shared views; one can fairly say that a significant part of what subsequently became known as Cartesian natural philosophy was plagiarized from Beeckman.

This is not to call into doubt Descartes' reputation as one of the most original thinkers of the seventeenth century, which, even leaving aside his seminal contributions to mechanics, optics and speculative metaphysics, would be guaranteed by his algebraic geometry alone. It is in any case a misunderstanding of originality to suppose it compromised by any elements anticipated in the views of others, as if it should be assessed in terms of the number of such elements rather than in the novel way these are related together. Descartes, of course, saw himself as free to appropriate propositions (*sententiae*) and even arguments from others without attribution, on the grounds that the status of these elements would be entirely different in a systematic chain of argument such as he was providing with his "order of reasons".[57] And it is, I think, beyond dispute that Descartes saw himself as providing a new and systematic foundation for the New Philosophy, based on a rationalistic metaphysics.

Nevertheless, it remains the case that Descartes was not shy to promote the merits of his system in terms of the wonderful new advances in knowledge it enabled, and even to appeal to this as a criterion of its success. And he presents as fruits of his new approach such fundamental features of his natural philosophy as the conservation of (quantity of) motion, the law of persistence in motion, the eschewing of internal motive force as cause of this persistence, and its replacement by God's direct action. Had Beeckman published his *Mathematico-physical Meditations* in 1630, Descartes would, of course, have lost his priority in print. But more than that, the fact that Beeckman had discovered this coherent ontology through piecemeal reflection on phenomena would have severely compromised Descartes' insistence on the necessity of the rational foundation he was trying to provide for natural philosophy, as well as exposing his own derivations of these results to a more skeptical scrutiny. From this perspective, I think the threat to Descartes' project as well as his reputation was very real.[58]

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[1] See Descartes' letter to Mersenne, 8th October 1629 (*Correspondance du P. Marin Mersenne*, v. 2, 1628-1630, ed. and annotated by Cornélis de Waard, Paris: Beauchesne et Fils, 1937; p. 301—hereafter *CMM* II). Mersenne

had received some challenge questions concerning the causes of consonance from Descartes, and had sent them on to Beeckman, only to hear from Beeckman that he himself was their original source; but when Mersenne reports this to Descartes, the latter interprets this as a boast on Beeckman's part "that he had been my master ten years ago", and abruptly breaks off all relations with him. He writes: "I am extremely obliged to you for bringing my friend's ingratitude to my attention. It is, I believe, the honour you did him in writing to him that has dazzled him, and he believed you would have a better opinion of him if he wrote to you that he had been my master ten years ago. But he is much deceived...".

[2] In this letter (AT I, 157 ff.) Descartes tells Beeckman (among other things) that "I have never learnt anything but idle fancies from your *Mathematical Physics"*, and that "you should not indulge your sickness by dwelling on the fact that I have sometimes accepted what you said, for it occasionally happens that even when the most incompetent person discusses philosophy, he says many things which by sheer chance coincide with the truth." (To Beeckman, 17 October 1630; *The Philosophical Writings of Descartes*, v. 3, ed. John Cottingham, Robert Stoothof, Dugal Murdoch and Anthony Kenny, Cambridge UP: Cambridge, 1991; 27—hereafter *CSM-K*). The immediate cause of this outburst was apparently Beeckman's writing to him, after a visit by Mersenne in summer 1630: "And since your Mersenne was immersing himself for whole days in my book manuscript, and seeing in it most of what you regard to be yours, and in the time devoted to those things was rightly in doubt concerning

their author, I explained what the facts of the matter were to him more freely perhaps than was pleasing to you or to him."

[3] A version of this planned work was published after Beeckman's death by one of his brothers, Abraham Beeckman, as D. Isaaci Beeckmanni, Medici & Rectoris Apud Dordracenos, *Mathematico-Physicarum, Meditationum, Quæstionum, Solutionum, Centuria* (Traiecti ad Rhenum, Apud Petrum Daniels Slost, 1644). See Klaas van Berkel, "Descartes' Debt to Beeckman", pp. 46-59 in *Descartes' Natural Philosophy*, ed. Stephen Gaukroger, John Schuster and John Sutton (London/New York: Routledge, 2000)—hereafter *DNP*.

[4] The full work that Descartes had almost ready for publication in 1633 included not only what was later published as the *Treatise on Light*, but also the *Treatise on Man* and material subsumed in the *Dioptrique* and *Metéorologie*

[5] van Berkel quotes John Smith of Nijmegen, who, on reading "the *Centuria Meditationum mathematico-physicarum* by the Dordrecht rector Isaac Beeckman (already written in 1629, but published only recently [by Beeckman's brother Abraham])", noted that it showed "that these [magnetic] corpuscles were not first thought of by Descartes"; "Descartes' Debt to Beeckman", p. 58.

[6] See the "Avertissement" by Charles Adam in *Œuvres de Descartes*, X 3-39 (hereafter cited as AT X, etc.); and Cornélis de Waard, *Journal tenu par Isaac Beeckman de 1604 à 1634*, 4 vols. (hereafter *JIB*, with volume and page), The Hague: M. Nijhoff, 1939-1953; and also the extracts contained in de Waard, *CMM* II (1937), n.1.

[7] Alexandre Koyré claims that what was decisive in "Descartes' attempt to rebuild physics on new foundations" was his decision to base it "on the 'order of reason' and not on the order of material substances", *Galileo Studies*, (transl. of *Études Galiléennes* by John Mepham; Atlantic Highlands, N.J.: Harvester Press, 1978); p. 89—hereafter *Galileo*.

[8] In his *Descartes' Metaphysical Physics* (hereafter *DMP*) Dan Garber wisely refrains from using the word 'inertia' in connection with Descartes' laws of motion: "They are more properly principles of persistence, principles that tell us what features of bodies God sustains in the world." (p. 203).

[9] In this vein, Descartes' biographer Geneviève Rodin-Lewis reckons Descartes' juvenile essays to be "more original than anything Beeckman had written" (*Descartes*, pp. 51, 131). But even so sympathetic an interpreter as John Schuster (who has done more than most to give Beeckman his due) writes that "Beeckman's very philosophical naïveté, his lack of concern to convince a cultivated audience, guaranteed that his fundamental beliefs and commitments were not masked under the weight of metaphysical and theological legitimation"; John A. Schuster, *Descartes and the Scientific Revolution, 1618-1634,* 2 vols., (hereafter *DSR*), Ann Arbor, Michigan, 1977; p. 58 (my thanks to Brian Baigrie for loaning me his copy of this work). This seems to imply both that Beeckman was naïve about theology—belied both by the fact that he had studied theology at Leiden between 1607 and 1610, and by what he does write in the *Journal* on theological issues—and that his theology

could only serve a legitimatory role for his fundamental beliefs, which I do not accept.

[10] H. Floris Cohen offers a plausible account along these lines, suggesting that Descartes' accusations about Beeckman's lust for praise "is really a classic example of psychological projection", since "the obsession with 'praise' and 'being taught' is clearly Descartes' own" (*Quantifying Music. The Science of Music at the First Stage of the Scientific Revolution, 1580-1650*. Dordrecht, 1984; 196). This interpretation is endorsed by Stephen Gaukroger in his impressive biography, *Descartes: an intellectual biography* (Oxford: Clarendon Press, 1995), who suggests further that "Beeckman had acted as a father figure for Descartes in 1618-19, and it is possible that his reaction to Beeckman may [have been] overdetermined by his relation to his father" (224).

[11] Beeckman, *JIB*, I, 132 (23rd December – 16th March), 1618; see also Descartes, AT X, 225; Koyré, *Galileo*, 117.

[12] "This decisive progress consisted in (a) the explicit assertion of the law of conservation of *motion*, which was thereby emancipated from the idea of *impetus*, and (b) the elimination of any *cause* within the moving body"; Koyré, *Galileo*, 119, n. 80.

[13] *JIB* I, 253, 23rd Nov.–26th Dec. 1618, interpolated text De Waard's. Beeckman had used his principle in support of the Copernican hypothesis earlier too: "Large bodies like the Earth and the planets, once moved, even in a plenum (that is, in the air or aether), move perpetually, or almost perpetually, because of their size (*magnitudine*)" (*JIB* I, 104, Feb. 6th–Dec. 23rd 1616).

[14] "The principle of continuous creation implies that no created thing can exist unless it is sustained by a creative force, and that every force that inheres in a thing is nothing other than that by which God puts it into existence at each instant." Martial Guéroult, "The Metaphysics and Physics of Force", in Stephen Gaukroger, (ed.), *Descartes: Philosophy, Mathematics and Physics* (Brighton, 1980), 197; quoted from Gaukroger, *Descartes*, 376.

[15] Descartes to Mersenne, 28th October 1640; AT III, 213; Gaukroger, *Descartes,* 371.

[16] See the essays by Stephen Gaukroger ("The foundational role of hydrostatics and statics in Descartes' natural philosophy," pp. 60-80) and John Schuster ("Descartes opticien: the construction of the law of refraction and the manufacture of its physical rationales 1618-1629", pp. 258-312) in *DNP*.

[17] This solution differs from the modern not only in the conception of a balance between centrifugal and centripetal forces, but also in regarding the *circular motion* as unaffected by either of these forces normal to it. Stephen Gaukroger infers from this that Descartes treats circular motion as inertial: "Since the motion that carries it along the circle AB is in no way impeded by the sling, this suggests that the motion is being treated as inertial. In other words, as well as accepting rectilinear inertia, Descartes accepts circular inertia" (*Descartes*, 246.) A more accurate conclusion, I propose, is that since there are no motions in a plenum that are unimpeded by extrinsic causes, strictly

speaking *there are no inertial motions in Cartesian physics*, even if certain motions are conserved under certain constraints.

[18] One might add that although rectilinear inertia is incompatible with circular inertia, it is not in fact incompatible with the conservation of circular motion of a body, whether in an orbit or about its own center. Beeckman is right that a rigid body set spinning about its own center in a vacuum will spin perpetually with the same motion; where he is wrong is in not seeing that a similar conservation of orbital circular motion is impossible if one rejects the idea (which Copernicus upheld) that the planets are attached to rigid spheres. What is required for such conservation in this case (as Hooke and then Newton will recognize) is a constant force directed towards the center.

[19] In his *Descartes*, Gaukroger argues that “we cannot take seriously the idea that [Descartes] might have arrived at his punctiform analysis of motion by reflecting on the nature of God’s activity, when his whole approach to mechanics from 1620 onwards has been in terms of instantaneous tendencies.” (248). From this he concludes that the foundation Descartes gives in the 1630s for his physics of instantaneous tendencies in terms of the constancy of God’s action should be construed as “legitmatory metaphysics” (13, 248. 292ff.) I do not follow him on this, but that is a topic for another time.

[20] Beeckman supplies the title: “A stone falling in a vacuum towards the center of the earth: how much its motion would increase at each moment, the account of Des Cartes.” (*JIB* I, 261)

[21] This is similar to the way in which the problem had been understood by Leonardo da Vinci. He also divides the time into “degrees of time” in each of which the heavy body “acquires a degree of movement (*grado di moto*) more than in the preceding degree of time, and similarly a degree of swiftness (*velocità*) greater than the degree of the preceding movement.” (M 44v; Marshall Clagett, *The Science of Mechanics in the Middle Ages*, Madison: University of Wisconsin Press, 1959; p. 572). Leonardo here concludes erroneously that “Therefore at each doubled quantity of time the length of the descent is doubled and also the swiftness of the movement.” But elsewhere he correctly concludes, as will Beeckman, that this discontinuous acquisition of *velocità* will give lengths of space “in arithmetical proportion” (M 47r-v; Clagett, p. 573.)

[22] According to Jean Buridan, “Impetus is a thing of permanent nature distinct from the local motion in which the projectile is moved...And it is probable that impetus is a quality naturally present and predisposed for moving the body in which it is impressed, just as it is said that a quality impressed in iron by a magnet moves the iron to the magnet.” *Questions on the Eight Books of Aristotle’s Physics*, VIII, Qu. 12, Marshall Clagett, p. 537. According to Nicole Oresme, “it may be said that it is a certain quality of the second species ... ; it is generated by motion by means of a mover, just as would be said of heat, when motion is the cause of heat” (*ibid.*, 552, my translation).

[23] Despite his otherwise perceptive criticisms of Koyré’s reading, John Schuster appears to agree with him on this, writing that while Descartes built “much of his natural philosophy and mechanistic optics ... around the analysis

of the magnitude and components of directional magnitude of the 'force of motion' possessed by a body at each moment of its motion," " by contrast, Beeckman always seems to have entertained a modern concept of motion just because he did not mention impressed or internal moving force" (*DSR*, I, 86-87).

[24] Beeckman, *JIB* I, 24-25: July 1613-April 1614. It is quoted in part by Koyré, *Galileo*, 117.

[25] For example, Beeckman writes of all the parts of a disintegrated body having "the same force of moving and resisting another as the whole", *JIB* III, 129; 13-30 September 1629.

[26] "Galileo gave up the idea of *impetus*, the internal cause of a body's motion. He does, of course, keep the word, but with a completely different meaning: instead of being the *cause* of motion, *impetus* becomes its *effect*. He saw clearly that *impetus*, if it is defined as the cause of motion, must be used up as it generates the motion. If it remains unchanged, this is because it plays no role at all in the continuation of the motion." (Koyré, *Galileo,* 75).

[27] Beeckman, *JIB* I, 265-266 (26th December, 1618). Here Beeckman equates *impetus* with the product of *corporeitas* and *celeritas*, i.e. heaviness and speed.

[28] "As for the abandonment of the idea of attraction, this is quite typical of Cartesian thought —Descartes manifestly prefers the idea of gravity to this obscure idea of action at a distance." (Koyré, *Galileo*, 122, n. 101).

[29] Descartes to Mersenne, 13th November 1629; *CMM* II, 316; *JIB* IV, 166.

[30] This is how Schuster reads the difference between Descartes and Beeckman on fall (*DSR*, I, 87).

[31] *Quod semel motum est, in vacuo semper moveri*. This is an exact replication of Beeckman's principle, *Quod semel movetur, semper movetur in vacuo* (*JIB* I, 263).

[32] Descartes to Mersenne, 18th December 1629, *CMM* II, 339-340; *JIB* IV, 170.

[33] Beeckman had written "at the third and fourth moments", clearly a mistake.

[34] Beeckman to Mersenne, 1st October 1629 (*CMM* II, 280-281; *JIB* IV, 160).

[35] Unaccountably, *CSM-K* has "twelve years ago" for Descartes' *ante undecim annos* (p. 16).

[36] "*Supponit, ut ego*"—an implicit denial of priority that must rank as one of the most disingenuous ever made in the history of physics. As de Waard observes, in his early work *Parnassus* (AT X, 219) "Descartes attributes the paternity of both the preceding hypotheses entirely to Beeckman, whom he designates as *vir ingeniosissimus*" (*CMM* II, 341, n. 2).

[37] Indeed, it is most likely that Beeckman showed Descartes his notebooks and discussed their contents with him when Descartes visited him

in Dordrecht at the end of 1628 and beginning of 1629. See Schuster, *DSR*, II, 566-79; see also Gaukroger, *Descartes*, 220.

[38] In his *Journal* entry of December 1618 (*JIB* I, 263), in his notes on Descartes' solution to his problem, Beeckman writes of "the Earth pulling by means of corporeal spirits"; this is, of course, consistent with his atomism.

[39] To Mersenne's objections Beeckman had replied: "You are not of my sentiment regarding the magnetic pull [*tractio*] of the Earth, and you judge that the gravity of the stone suffices for it to descend. However, I also investigate the cause of gravity. For if a stone is set above the Earth outside the domain of its activity, it will rest in the vacuum no differently than the Earth itself together with its surrounding air, as though it were a second little Earth..." (Beeckman to Mersenne, 30th April 1630, *JIB* IV, 184).

[40] Marin Mersenne, *Harmonie Universelle*, I, bk. 3 (Paris 1636), 206; *JIB* IV, 171. Beeckman himself, in his letter to Mersenne of 1st October 1629, had rendered his account of fall in terms of the "pulling force" (*vis trahens, tractio*) of the Earth. If this were switched off in the second and third moments, "no one would deny that the same thing would happen to this stone as a result of this force as would happen in violent motion, that is, it would continue to move for as long as the stone is projected... Therefore at the second moment it is moved partly by the motion remaining from the first moment, partly also by the new motion of the second moment." (*CMM* II, 280-281; *JIB*, IV, 160).

[41] See Clagett, "The Manuscripts of Leonardo da Vinci", pp. 572-75 in *The Science of Mechanics in the Middle Ages* (Madison: University of Wisconsin Press, 1959): "the aforesaid powers (*potentiae*) are all pyramidal since they commence in nothing and proceed to increase in degrees in arithmetical proportion (*gradi di proportione aritmetricha*)" (M 44r; p. 572). Leonardo seems to be in error in writing "pyramidal", since his figure is clearly a triangle; unless the force or power increases both as the speed and the time, in which case we have the second "more difficult" case considered by Descartes above. But Leonardo, unlike Descartes, gives triangular, not pyramidal, proportions for this case.

[42] Drake (following Marshall Clagett) erroneously attributes Descartes' mathematical move to the limit to Beeckman, commenting that it could be allowed "if we allow the kind of reasoning by which one could also prove that the diagonal of a square equals the sum of two sides of it" (Stillman Drake, *History of Free Fall*, pp. 71-72). But this sarcasm is unwarranted; the length of the perimeter along the protruding squares remains the same as one continually halves their widths and doubles their number, whereas their areas decrease in proportion to their widths. Descartes' proof involves a sound anticipation of the operation of taking the limit.

[43] The "somewhat less than" is my interpolation, and seems necessary for the correct sense.

[44] De Waard cites the following passage, which I have translated from the Flemish and Latin: "All things go by jerks (*horten*), as one can see when

heavy weights are moved slowly by machines. Hence it can be proved that everything does not consist in [parts] divisible to infinity" (*JIB* III, 348; 2nd May–7th July 1634). I take this to mean that motion increases by discrete increments as a result of discrete pushes or pulls: thus both force and accelerated motion accrue discontinuously, dividing physical space and time into physical indivisibles.

[45] I was unable to find this passage, quoted from *CMM* II, 123, in Beeckman's *Journal*. But compare: "God first moved atomic bodies no less than he created them; once moved, they never came to rest, unless by colliding with one another" (*JIB* I, 132; 23rd December 1616–March 16th 1618); "Besides, many Earths could have been naturally constituted by God in this world, with each one conserving any motion perpetually" (*JIB* II, 232; Jan 22nd–Feb. 21st 1623).

[46] Here I agree with Dan Garber, who writes that although "Descartes was certainly introduced to the impetus theory in his years at La Flèche, ... it was almost certainly Beeckman who introduced Descartes to the somewhat different principle he had discovered for himself" (*DMP*, 10).

[47] Descartes would at least have read the Coimbrian Fathers' commentary on Aristotle's *Physics*. Daniel Garber quotes them as saying that the cause of a body's remaining in motion was "a certain force or impetus impressed by the hurling of the moved object, which inheres in it" (*DMP*, 226).

[48] Cf. *JIB* II, 99, and Schuster's remark: "Beeckman produced no textbook on natural philosophy. He actually prided himself on the spontaneous

character of his meditations, which he thought offered a more genuine insight into the questions posed than any pre-arranged program of research." (*DSR*, I, 57).

[49] Schuster writes: "Surely (though it is obviously hard to demonstrate) Beeckman's piecemeal, inconclusive approach to speculation only reinforced Descartes' instinct for method, order and style" (*DSR*, I, 71).

[50] In his letter to Mersenne of 18th December 1629 Descartes had written: "*Oportet meminisse nos supponere illud quod semel motum est, in vacuo semper moveri, et in meo tractatu demonstrare conabor*" (AT 1, 90; CSMK 15; *CMM* II, 340).

[51] Beeckman, *JIB* II, 246; 16th April–16th July 1623; I have substituted "assert (*asserare*)" for his "deny (*negare*)" in order to preserve the sense. Beeckman refers to his principle as a theorem in late 1618 (*JIB* I, 256), and in early 1629 he claims to have proved it: "For we have proved before that *what is once moved in a vacuum moves always*" (*JIB* III, 117).

[52] Olivier Bloch, *La philosophie de Gassendi* (The Hague, 1971), pp. 215-16. As Bloch notes, this aspect of Epicureanism was known and emphasized by Gassendi. He quotes Gassendi's comment in (Tours 709 folio 185r): "Epicurus believes all atoms to be endowed with a certain internal energy, ôr inborn vigor, by which they set themselves in motion" (Bloch, 215, n. 55). See also the discussion in Gassendi's *Animadversiones in decimum librum Diogenis Laertii* (Lyons, 1649; reprint ed. N.Y./London: Garland), p. 445.

[53] Cf. Garber, *DMP,* 220: "Though a body would go straight if it were not interfered with, other bodies are always interfering with it."

[54] "It is obvious why Descartes chose to express this law in terms of tendencies rather than more straightforwardly in terms of a state of a body that persists conditional[ly] on a lack on interference. In his plenum, this condition of noninterference can *never* be met." (Garber, *DMP*, 220).

[55] For an illuminating discussion of this point, see Garber, *DMP*, 204-230, esp. 214ff.

[56] See Stephen Gaukroger, "The foundational role of hydrostatics and statics in Descartes' natural philosophy," *DNP*, 60-80. See also Gaukroger and Schuster, "The Hydrostatic Paradox and the origins of Cartesian dynamics", *Studies in the History and Philosophy of Science* 33 (2002), 535-572; pp. 555-6 contain a succinct account of Beeckman's rules.

[57] Thus Descartes says (AT X, 204), "As we cannot … complete a sentence unless it consists of words that are in the dictionary, so neither [can we compose] a book except out of the sentences [or opinions, *sententiae*] that are found in others. But if the things I say are so coherent among themselves and so connected that they follow from each other, then it will not follow from this argument that I have borrowed my opinions [*sententiae*] from others any more than that I have taken the words themselves from the dictionary". I have quoted this passage in Stephen Menn's translation from his *Descartes and Augustine* (Cambridge: Cambridge University Press, 1998) pp. 12-13. Menn argues against the apologetics of Gilson, Gouhier and Guéroult, who tried to

downplay Descartes' debt to Augustine, that "Descartes took the central concepts of his metaphysics from Augustine or Augustinians" as a foundation for his new mechanical physics, perhaps altering it in the process (16).

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