Leibniz's Inversion of Zeno:

Continuity of Motion, Substantial Action and Plurality[1]

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Itaque actio in corpore non nisi per aversionem quandam intelligi potest. Si vero ad vivum reseces, seu si momentum unumquodque inspicias, nulla est. Hinc sequitur Actiones proprias et momentaneas, earum esse rerum quae agendo non mutantur. (A VI.iii 566)[2]

In what follows I shall attempt to shed light on both the main themes of this volume by considering the relationship of Leibniz's views to Zeno's Paradoxes. I do not mean to suggest that Leibniz formulated his views as explicit responses to Zeno of Elea's famous arguments against motion and plurality (it is probably more accurate to see him as responding to the whole tradition of thought prompted by them, from Plato and Aristotle through Sextus Empiricus and the Scholastics to Galileo Galilei and the moderns). Nevertheless I believe a direct comparison of the two is informative. For I think that in his treatment of the problems of continuous motion and plurality Leibniz employs a characteristic style of arguing—an argument schema—which can usefully be regarded as an inversion of Zeno's typical way of reasoning. In this paper I shall attempt to distinguish several manifestations of this argument schema in the evolution of Leibniz's thought over the years, culminating in his mature philosophy of substance (which I take to be in place, barring fine tuning in the exposition, by the early 1680's).

In concentrating on this recurring schema I risk minimizing the changes that occurred in Leibniz's thought on these matters. These are both profound and manifold, but are too complex for me to trace here in anything but the most oblique fashion. Leibniz's own path through the continuum is as much a labyrinth as the problem itself, as I can attest

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[2] My abbreviations for primary texts are as follows: A = *Gottfried Wilhelm Leibniz: Sämtliche Schriften und Briefe*, ed. Berlin: Akademie-Verlag, 1923-96—the so-called Academy Edition; references to it specify series, volume and page, e.g. A VI ii 229, or series, volume and piece number, e.g. A II i N68, or both, A VI iii N2: 28. AT = *Oeuvres de Descartes*, ed. Ch. Adam & P. Tannery, (Paris: J. Vrin, 1964-76); references are to volume and page, e.g. AT VIII.1, 71. G = C.I. Gerhardt, ed., *Die Philosophische Schriften von Gottfried Wilhelm Leibniz* (Berlin: Weidmann, 1875-90; reprint ed. Hildesheim: Olms, 1960), 7 vols.; references are to volume and page, e.g. G VI 264. All translations are my own.

from my experience of collecting together his manuscripts bearing on this topic from the 1670s and 1680s.[3] Yet Leibniz left us with only some enigmatic formulations of the results of his labours, statements which require considerable work on the part of the interpreter to decipher.[4] Moreover they leave unexplained the relationship of his views on substance and reality to many elements that were essential to his thought on the continuum: in particular, his nuanced and original treatment of the infinite, his philosophy of mathematics and the status of infinitesimals, his physics of matter (the fluid and the firm, atoms and the void, elasticity), his doctrine of force, his principle of continuity, and above all, the dynamical aspect of the problem of the continuum.

It is this last aspect that I want to concentrate on here. Or rather, by making it central, I hope to cast light on how Leibniz's doctrine of substance is rightly regarded by him as a pivotal element in his solution to the continuum problem. For I shall argue that the main thrust of his inverted version of Zeno's Dichotomy is that motion cannot be reduced to the covering of an interval of space in an interval of time, but must be founded in a substantial action (*conatus*, active force or appetition). I also argue that a comparison of Leibniz's views with Eleatic arguments against plurality shows the same inversion of Zeno's argument schema, and that a proper appreciation of this argument from matter to a plurality of immaterial substances shows the indispensability of corporeal substance to Leibniz's system.

Zeno's Dichotomy

First, let me begin with Zeno's argument against the possibility of motion known as the Dichotomy (also known as the "Racecourse", or paradox of the half-distances). Aristotle

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[3] G. *W. Leibniz: The Labyrinth of the Continuum: Writings from 1672 to 1686*, ed. and trans. R. T. W. Arthur (forthcoming with Yale University Press, 2001).

[4] A good example of such an attempt is Pauline Phemister's fine article in this volume. See also Enrico Pasini, *Il Reale e L'Immaginario* (Turin: Sonda, 1993), pp. 193-210.

could hardly have stated it more tersely: "The first [of Zeno's arguments against motion] says that there is no motion, because the moving body must reach the midpoint before it gets to the end" (*Phys*. 239b 11).[5] But we are given a more complete rendering of Zeno's reasoning by the Neoplatonic commentator Simplicius:

> If there is motion, the moving object must traverse an infinity in a finite [time]; and this is impossible. Hence motion does not exist. He demonstrates his hypothesis thus: The moving object must move a certain stretch. And since every stretch is infinitely divisible, the moving object must first traverse half the stretch it is moving, and then the whole; but before the whole of the half, half of that, and again, the half of that. If then these halves are infinite, since, whatever may be the given [stretch] it is possible to halve it, and [if, further] it is impossible to traverse the infinity of these stretches in a finite time ... it follows that it is impossible to traverse any given length in a finite time.[6]

.... ❻ ❺ ❹ ❸ ❷ ❶

That is, in order to reach the point ❶ it must first reach the halfway point ❷, and before that ❸, and so on *in infinitum.* Therefore no finite stretch can be completed if an infinity of its subintervals cannot be. Here I take the reference to a finite time to be inessential to the gist of the paradox. We can agree with Aristotle that if the body moves at a constant speed, the time will be divisible into just as many subintervals each of which is in the same proportion to the corresponding subintervals of the distance. But the gist of the paradox is that since a half of each subinterval must be traversed before the whole of it, by dichotomy *in infinitum*, the motion can never get started. Therefore there is no motion.

Leibniz's Inverted Dichotomy 1: Indivisibles

If we now turn to the *Fundamenta Praedemonstrabilia* of the *Theoria Motus Abstracta* (TMA) which Leibniz sent to the Academie Française in 1671 (A VI ii N41), we

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[5] Quoted from Gregory Vlastos, "The Eleatics", *Philosophical Classics*, ed. Walter Kaufmann (Prentice-Hall, 1968), pp. 22-33: 29. This is the fullest of Aristotle's four references to the paradox, all given here by Vlastos; the others are at *Topics*, 160b 7, *Phys.,* 233a 21, and *Phys.,* 263a 5.

[6] *Phys*. 1013, 4ff.; also quoted from Vlastos, op. cit.

find a beguilingly similar argument. For Leibniz too asserts that any motion must have a beginning. But he assumes that there *is* motion, and by this means inverts the dichotomy to establish something quite different. This is the first avatar[7] of what I call Leibniz's Inverted Dichotomy:

> (4) *There are indivisibles ôr unextended things*, otherwise neither the beginning nor the end of a motion or body is intelligible. This is the demonstration: any space, body, motion and time has a beginning and an end. Let that whose beginning is sought be represented by the line *ab*, whose midpoint is *c*, and let the midpoint of *ac* be *d*, that of *ad* be *e*, and so on. Let the beginning be sought to the left, on *a*'s side. I say that *ac* is not the beginning, since *dc* can be taken away from it without destroying the beginning; nor is *ad*, since *ed* can be taken away, and so on. Therefore nothing from which something on the right can be taken away is a beginning. But that from which nothing having extension can be taken away is unextended. Therefore the beginning of a body, space, motion, or time (namely, a point, an endeavour [*conatus*], or an instant) is either nothing, which is absurd, or is unextended, which was to be demonstrated. (A VI ii N41: 264)[8]
>
> *a e d c b*

There is much to say about this argument, but I shall try to restrict myself to what is relevant here. First, to state the obvious: body, space, motion and time are all continua, which is why Leibniz represents them by a line. Second: like Ockham before him,[9] Leibniz insists that the continuum is not merely infinitely divisible in this way, but *actually infinitely divided*. Thus Aristotle's way of refuting Zeno's dichotomy paradox by saying that the subintervals and divisions are alike merely potential, is precluded for him. Third, the "points" or "indivisibles" mentioned here are not to be confused with Euclidean

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[7] That is, it is the first avatar I shall consider; there are many others in Leibniz's writings that I do not have the space to discuss here. In using the term avatar, I acknowledge a debt to the fine essay by Jorge Luis Borges, "Avatars of the Tortoise", pp. 202-208 in *Labyrinths*, ed. Donald A. Yates and James E. Irby (New Directions Press, 1964). Borges takes the Achilles for his paradigm (although he notes that the mechanism for the Dichotomy is almost identical) and traces its avatars through Aristotle, Hui Tsu, Aquinas, Lotze, Bradley, and James.

[8] I give an English translation of all of the *Fundamenta Praedemonstrabilia* in an Appendix of my translation volume. The *ôr* denotes an "or of equivalence" (*seu* or *sive*). Although I was obliged to abandon this notation in the final version of the volume, its use in earlier drafts explains its occurrence in articles by myself and Sam Levey based on those translations.

[9] Philip Beeley has drawn attention to Ockhamist precedents of Leibniz's doctrine of actually infinite division. See his *Kontinuität und Mechanismus* (Stuttgart: Franz Steiner, 1996), especially pp. 56-66.

points. Euclid defines a point as "that which has no part"; Leibniz calls a point in this sense a *minimum*, and claims that there are no minima in either space or body.[10] As Leibniz defines it in the TMA, however, a point is an *unassignable* (*inassignabile*), i.e. something "smaller than can be expressed by a ratio to another sensible magnitude unless the ratio is infinite" (A VI ii 265). Fourth, these unassignables may nonetheless stand in a finite ratio to one another. The endeavours in a given instant of two unequal motions will be in the ratio of the motions themselves; and the respective points traversed by the moving bodies in that instant will be in the same proportion.[11] Fifth, Leibniz sees the proof of indivisibles as crucial to "the true distinction between bodies and minds" (A VI ii 266): for "thought consists in endeavour, as body consists in motion" (A II i 173), and since endeavour cannot last in a body for more than a moment, the mind of a body is momentary, as opposed to true minds, which are able to sustain and compare endeavours.[12]

To recap: Zeno had argued in his Dichotomy that since one could not move across any interval without first crossing the first half of this, and before this the first half of this, *ad infinitum*, there can be no beginning of any motion; therefore motion is impossible. Leibniz inverts this reasoning, arguing that since motion is real and must therefore have a beginning, and since (by the Dichotomy) this beginning cannot be found in any extended subinterval of the continuum, each and every subinterval of the continuum must have an unextended beginning. (The same argument will apply to body, time and space, assuming they are real.)

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[10] This is reminiscent of the doctrine that Aristotle ascribes to Plato: "Plato even used to fight against this class of things [sc. points] as being something that geometers believed in, whereas he called indivisible (*atomoi*) lines the origin of the line, and this he often postulated" (*Metaphysics* I 9, 992a19-22); quoted from Richard Sorabji, "Atoms and Time Atoms", pp. 37-86 in *Infinity and Continuity in Ancient and Medieval Thought*, ed. Norman Kretzmann (Ithaca: Cornell University Press, 1982): p. 47. Whether Leibniz is knowingly advocating a Platonist position here is an intriguing question.

[11] The comparability of the indivisibles is guaranteed by their being generated in equal instants by moving points with proportional velocities (A VI ii 266). Following Hobbes, Leibniz correctly discerns the similarity with Cavalieri's method of indivisibles, where only indivisibles generated by the *transitus* or passage of the same *regula* may be compared. For the latter, see Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles* (Bologna, 1980), and Kirsti Andersen, "Cavalieri's Method of Indivisibles," *Archive for History of Exact Sciences* 31, 4, pp. 291-367.

[12] See Daniel Garber, "Motion and Metaphysics in the Young Leibniz," in Michael Hooker, ed., *Leibniz: Critical and Interpretive Essays* (Minneapolis: University of Minnesota Press, 1982).

An important corollary of this is that every continuum must contain an actual infinity of unextended beginnings (endeavours, points or instants). That is, these beginnings do not just mark potential cuts in the continuum, as they would if it were merely potentially or indefinitely divided. For Leibniz rejects Descartes' "indefinite division", insisting that the continuum is actually infinitely divided. Nevertheless, Leibniz does not claim that the continuum is composed of these indivisibles. The intervals are the parts of the continuum, and the indivisibles are their beginnings and ends.[13]

But at this point we must raise a question of consistency. If Leibniz's *inassignabilia* are the endpoints of intervals, what arguments can he give against minima that would not also apply to his indivisibles? In the TMA Leibniz rejects minima ("things which have no magnitude or part") on two grounds: first, such things would "have no situation", "since whatever is situated somewhere can be touched by several things simultaneously that are not touching each other," something impossible for such partless points; and second, "a minimum cannot be supposed without it following that the whole has as many minima as the part, which implies a contradiction (A VI ii 264). The first objection is taken straight from Plato's *Parmenides* (138a), and I will not comment further on it here[14]; but the second, as we shall see, depends not on whether the points have any internal structure, but only on the fact that they are endpoints of subintervals of a line. Thus, since this is an essential feature of Leibniz's indivisibles in the TMA, it should preclude them too.

Interestingly, when Leibniz explicitly lays out this second argument against minima in a piece written in Paris two years later (*On Minimum and Maximum; on Bodies and Minds*: A VI iii N5, November 1672-January 1673), he has come to see that it will indeed rule out his indivisibles just as surely as minima: it is headed "*There is no minimum, ôr indivisible, in space and body*" (A VI iii N5: 97). The argument's main premise is that each point contained in a line may be conceived as the endpoint of another line cutting it

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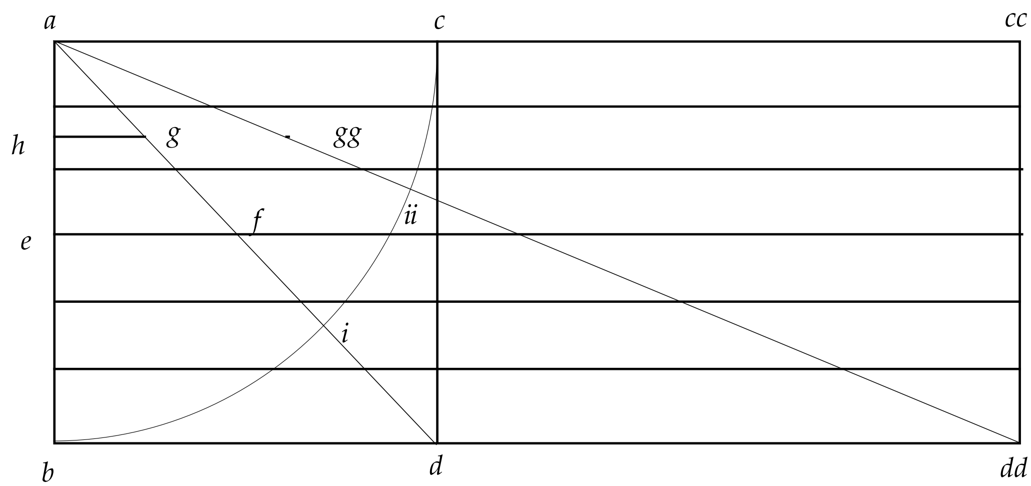
[13] Here I demur with what I wrote in my forthcoming translation volume (see fn. 3), where I imply that Leibniz only came to reject the composition of the continuum out of *inassignabilia* in his *De minimo et maximo* of Winter 1672-73. One should note, however, that Leibniz does explicitly uphold such a composition in his *Demonstratio Substantiarum incorporearum* of Fall 1672, albeit in a cancelled draft.

there: "every indivisible point can be understood as the indivisible boundary of a line". From this and the fact that any given line *ab* can be indefinitely divided, it follows that "if there is [an indivisible] in the line *ab*, then there will be one in it everywhere". Leibniz's argument (which I have elsewhere dubbed "Leibniz's Diagonal Argument", even though it is not original with him) then proceeds as follows:



> So let us understand infinitely many lines parallel to each other, and perpendicular to *ab*, to be drawn from *ab* to *cd*. Now no point can be assigned in the transverse line or diagonal *ad* which does not fall on one of the infinitely many parallel lines extending perpendicularly from *ab*. For, if this is possible, let there exist some such point *g*: then a straight line *gh* may certainly be understood to be drawn from it perpendicular to *ab*. But this line *gh* must necessarily be one of all the parallels extending perpendicularly from *ab*. Therefore the point *g* falls—i.e. any assignable point will fall—on one of these lines. Moreover, the same point cannot fall on several parallel lines, nor can one parallel fall on several points. Therefore the line *ad* will have as many indivisible points as there are parallel lines extending from *ab*, i.e. as many as there are indivisible points in the line *ab*. Therefore there are as many indivisible points in *ad* as in *ab*. (A VI iii N5: 97)

Now Leibniz assumes "in *ad* a line *ai* equal to *ab*". Since *ai* is equal to *ab* it will contain the same number of points (which could be established by drawing infinitely many straight lines parallel to one drawn from *i* to *b*). But

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[14] See G. E. L. Owen, "Zeno and the Mathematicians" (reprinted in Wesley Salmon, ed., *Zeno's Paradoxes*, Bobbs-Merrill, Indianapolis & New York, 1970, pp. 139-163) for a discussion of the impact of this argument on Aristotle's treatment of points and lines in his *Physics*.

> since there are as many points in *ai* as in *ab* ... and as many in *ab* as in *ad*, as has been shown, there will be as many indivisible points in *ai* as in *ad*. Therefore there will be no points in the difference between *ai* and *ad*, namely in *id*, which is absurd. (A VI iii N5: 97-98)

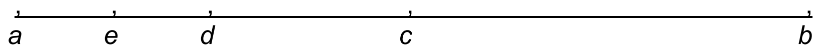
Therefore, Leibniz concludes, there are no indivisible points (points that could be conceived as boundaries of a line) in any line, and therefore none in space or body. Now it follows from this by another application of the dichotomy that "*There is no minimum or indivisible in time and motion*" (98) either.

> For let us suppose a space *ad* is traversed with a uniform motion in a time *ab*. Then in half the time *ae* half the space *af* will be completed, and in a thousandth of the time, a thousandth of the space, etc. Therefore in an indivisible of time, an indivisible of space will be traversed, since time and space are divided proportionately. For let us suppose that in a minimum of time the amount of space traversed is not a minimum: then in a time however small, provided it is not a minimum, infinite divisible spaces would be traversed, and in some perceptible time, an infinite space would be traversed. For the ratio of an indivisible—if such a thing is understood to exist—to the divisible, or the ratio of the minimum in the continuum to whatever is not a minimum, is that of the finite to the infinite. (A VI iii N5: 98)

Inverted Dichotomy 2: Primacy of Endeavour

Now from all this it might be inferred that Leibniz has found himself forced to give up not only his theory of indivisibles of the TMA, but also everything that depended on it, including his theory of cohesion and the reality of motion itself. For if there are no beginnings of motion, then his attempt to save its reality from Zeno's Dichotomy appears to have failed. But to infer this would be premature: Leibniz does not abandon his "beginnings" of motion. Indeed, immediately after the above proof in 1672/3, he reiterates his Inverted Dichotomy to prove the existence of infinitely small beginnings of lines and motions:

> *There are some things in the continuum that are infinitely small, that is, infinitely smaller than anything given that is sensible.*
> First I show this for the case of space as follows. Let there be a line *ab*, to be traversed by some motion. Since some beginning of motion is intelligible in that line, so also will be a beginning of the line traversed



> by this beginning of motion. Let this beginning of the line be *ac*. But it is evident that *dc* can be cut off from it without cutting off the beginning. And if *ad* is believed to be the beginning, from it again *ed* can be cut off without cutting off the beginning, and so on to infinity.

(Here Leibniz waxes poetic:)

> For even if my hand is unable and my soul unwilling to pursue the division to infinity, it can nevertheless in general be understood at once that everything that can be cut off without cutting off the beginning does not involve the beginning. And since parts can be cut off to infinity (for the continuum, as others have demonstrated, is divisible to infinity), it follows that the beginning of the line, i.e. that which is traversed in the beginning of the motion, is infinitely small. (A VI iii 98-99)

By reiterating this argument, Leibniz shows that he is not prepared to give up the reality of motion at this juncture. Nor indeed will he ever give up the reality of motion in this sense. He upholds it explicitly in a dialogue (which we shall shortly be considering) whose conclusion is that there is no such thing as uniform continuous motion; and he upholds it in his exchange with Sturm, who advocates a continual recreation, and in the heat of his controversy with the Newtonians, where he is trying to convince them that absolute motion is imaginary.[15] But to return to his argument in the Winter of 1672/3: how can Leibniz maintain that there are infinitely small beginnings of motion if there are no indivisibles? He is perfectly conscious of the difficulty: indeed he now repeats the Inverted Dichotomy to undermine his previous identification of unextended points as infinitely small lines in a space conceived as pre-existing:

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[15] "*Pacidius*: Still, we know that a place is traversed by a moving body, that is, that there is some motion./*Charinus*: This is what we experience, certainly, and it is not our place to call into question the reliability of the senses and to doubt the reality of motion." (A VI iii 556; 1676). "Every individual substance acts without interruption, not excepting body itself, in which no absolute rest is ever to be found" ("On Nature Itself", (G IV 504-16; 1698). "I grant that there is a difference between an absolute true motion of a body and a mere relative change of its situation with respect to another body... 'Tis true that, exactly speaking, there is not any one body that is perfectly and entirely at rest..." (Leibniz's Fifth Paper to Clarke, §53; 1716). Leibniz claims that although relative motion is not wholly real but phenomenal, what is real in motion is the active force in it responsible for all its changes (cf. A VI iv 279).

> *There is no space without body, and no body without motion.*
> This wonderful method of demonstration, unnoticed by anyone else, became clear to me from a more intimate knowledge of indivisibles. For I shall show that if there is some space in the nature of things distinct from body, and if there is some body distinct from motion, then indivisibles must be admitted. But this is absurd, and contrary to what has been demonstrated. The consequence is proved as follows. Suppose we understand a point as an infinitely small line, there being one such line greater than others, and this line is thought of as designated in a space or body; and suppose we seek the beginning of some body or of a certain space, i.e. its first part; and suppose also that anything from which we may cut off something without cutting off the beginning cannot be regarded as the beginning: with all this supposed, we shall necessarily arrive at indivisibles in space and body. For that line, however infinitely small it is, will not be the true beginning of body, since something can still be cut off from it, namely the difference between it and another infinitely small line that is still smaller; nor will this cease until it reaches a thing lacking a part, or one smaller than which cannot be imagined, which kind of thing has been shown to be impossible. (A VI iii 99-100)

Since this *consequentia* is absurd, there can be no "space in the nature of things distinct from body" and no "body distinct from motion".

> But if a body is understood as that which moves, then its beginning will be defined as an infinitely small line. For even if there exists another line smaller than it, the beginning of its motion can nonetheless be taken to be simply something that is greater than the beginning of some other slower motion. But the beginning of a body we define as the beginning of motion itself, i.e. endeavour, since otherwise the beginning of the body would turn out to be an indivisible. Hence it follows that *there is no matter in body distinct from motion*, since it would necessarily contain indivisibles. (A VI.iii 100)

From here Leibniz proceeds in a crescendo of enthusiasm to the conclusions that "to be a body is nothing other than to be moved," that motion and body are nothing more than "being sensed by some mind," and that this requires "some mind immune from body, different from all the others we sense," namely God (A VI iii 101). Interesting though these claims are, I cannot pursue them further here. The point I wish to stress is that with this argument Leibniz insists on the *ontological primacy of endeavour*: what is real in motion at an unassignable time is more fundamental than body itself. Body is nothing except insofar as it is defined by an endeavour; and space and time have no reality in themselves, in abstraction from the bodies and motions individuating them.

That this doctrine is no passing whim is shown by his appeal to it in an exchange with Malebranche in 1675. Malebranche, arguing in favour of the Cartesian identification of matter with extension, had tried to persuade Leibniz that an extended void would have distinct parts, which would therefore be separable and movable, and therefore be parts of matter.[16] In rejecting this, Leibniz had declared his belief that "it is necessary to maintain that the parts of the continuum exist only insofar as they are effectively determined by matter or motion" (Letter to Malebranche, March-April 1675 (?): G I 322; Malebranche, *Oeuvres*, 97).

How then is body defined by motion? Without going into the details, the gist of the idea is that bodies are individuated by sharing a motion or endeavour in common. On the face of it, this is the same as Descartes' view, where a body is individuated by its own proper motion, so that all its parts, in sharing this motion, are relatively at rest (even though they may also possess differing proper motions of their own).[17] Similarly, according to Leibniz's theory,

> It is manifest that a body is constituted as definite, one, particular, distinct from others, by a certain motion or particular endeavour of its own, and if it is lacking this it will not be a separate body, but [there will be] one continuous body cohering with it by whose motion alone it is moved. And this is what I have said elsewhere, that cohesion comes from endeavour or motion, that those things which move with one motion should be understood to cohere with one another.[18]

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[16] See the exchange in G I 321-327, reproduced in Malebranche, *Oeuvres Complètes*, Tome XVIII: *Correspondance et Actes 1638-1689*, ed. André Robinet, Paris: J. Vrin, 1961, pp. 96-104.

[17] "By a 'body' or a 'part of matter' I understand everything that is transferred together, even though this may consist of many parts which have different motions relative to one another" (Descartes, *Principles of Philosophy,* II, §25; AT VIII.1 53-54). I believe this definition of body in terms of motion in common is of the greatest relevance to the mereological issues raised by Sam Levey and Glenn Hartz in this volume; but unfortunately I do not have the space here to argue the point.

[18] Proposition 14, *Propositiones Quaedam Philosophicae*, (A VI iii N2: 28). Leibniz wrote this tract, probably intended for publication, in early-to-mid-1672.

The difference is that on Descartes' view mutual rest can be the cause of the cohesion, something Leibniz emphatically rejects. According to his theory, it is the endeavour of a given body to enter the place of another that is responsible for their cohering together as a continuous whole; for as it endeavours to enter, it will occupy a greater point, and therefore encroach on the other body.[19] Thus it is by virtue of occupying two places at once (its own and a vanishingly small part of the other body's place) that one body coheres with another.[20] Moreover, according to Leibniz no body is ever completely at rest, since all endeavours persist even when their observable effects are masked by the composition of motions.[21]

But a decisive break with all this occurs in the spring of 1676 with Leibniz's rejection of infinitely small actuals. By early April he is claiming to have shown that the only unassignable is an endpoint, a true minimum, and that the differentials of his calculus are not infinitely small "but nothing at all."[22] Leibniz does not tell us where he proves this or how. I have surmised that it might have been an application of what he later calls his "Herculean argument"—that "all those things which are such that it is impossible for anyone to perceive whether they exist or not, are nothing"—to his infinitesimal

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[19] Further analysis of this theory can be found in my "Cohesion, Division and Harmony: Physical Aspects of Leibniz's Continuum Problem (1671-1686)," *Perspectives on Science,* **6**, nos. 1 & 2, 110-135, 1999.

[20] "*If one body endeavours to move into the place of another, these two bodies are continuous.* Endeavour is the beginning of motion at a given moment. Therefore it is the beginning of a change of place, i.e. of a transition from place to place, and therefore is in both places at the same time, since it cannot be in neither, i.e. nowhere." (transl. from *De Consistentia Corporum*, A VI iii N4: 95-6).

[21] "No endeavours die away, but all are in general *efficacious and perpetual*, even though they cannot be sensed, having been mixed up with the other endeavours added on top of them, with lines of motion varied beyond measure by such a manifold composition" (A VI iii N4: 95).

[22] "[A] differential is not infinitely small, but that which is nothing at all" (A VI iii N52: 434; March 26); "[We] have supposed a point to be that whose part is nothing, an extremum; for we have already shown that there is nothing else unassignable"; "the unbounded, i.e. that which is greater than anything finite, is something, and the infinitely small is not" (A VI iii N69: 498, 502; c. April 10).

differences (cf. Newton's *Lemma 1* of his Method of First and Last Ratios).[23] At any rate, from now on points are mere endpoints, but not infinitesimals, and endeavours are no longer infinitely small motions.[24] A body is still regarded by Leibniz as individuated by the motion its parts share in common, and as containing other parts moving with their own diverse motions *in infinitum*. The difference is that now the actual infinite division is understood as containing no smallest part: just as the converging infinite series summed by Gregory of St. Vincent contain an infinity of terms, but no smallest, so a finite body can contain an infinity of subdivisions without this entailing that it is resolved into minima.[25]

It is these conclusions that form the context for the dialogue on continuity Leibniz pens in November of the same year (1676). For in this dialogue, *Pacidius Philalethi* (A VI.iii N78), Leibniz examines the question of the continuity of motion without presuming the existence of unassignables or endeavours. Indeed, here he is explicit that, in stark contradiction to his theory of cohesion based on infinitesimal endeavours, no body can ever be in two places at one moment.[26] Let us now turn to his arguments there.

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[23] In *Corpus non est substantia* (A VI iv N316: 1637) Leibniz argues that application of this "Herculean argument" will put an end to controversy over the existence of the infinitely small. This is presumably because the latter is usually defined as a difference smaller than any that can be assigned or perceived. Where the Akademie editors date this piece from about 1689, I think it might have been written some ten years earlier. At any rate, by January 1677 he was already applying the principle that "whatever cannot be distinguished, not even by someone omniscient, is nothing" to refute the existence of motion in a vacuum (A VI iv N360: 1971).

[24] In his *De Motu et Materia*, written April 1-10, Leibniz claims he has "demonstrated elsewhere very recently that endeavours are true motions, not infinitely small ones" (A VI iii N68: 492).

[25] "Accordingly, being divided without end is different from being divided into minima, in that [in such an unending division] there will be no last part, just as in an unbounded line there is no last point" (A VI iii N71: 513). See also (A VI iii N69: 498), and (A VI iii N78: 565-6): "There is no portion of matter which is not actually divided into further parts... This does not mean, however, either that a body or space is divided into points, or time into moments, because indivisibles are not parts, but the extrema of parts; which is why, even though all things are subdivided, they are still not resolved all the way down into minima".

[26]26 "[A body in instantaneous motion] will either be nowhere or in two places, the one it leaves and the one it acquires, which is no less absurd than what you have shown, that it simultaneously is and is not in some state" (A VI iii N78: 545).

Inverted Dichotomy 3: No State of Action in Body

In the preamble to the dialogue Leibniz refers to his previous difficulties with “beginnings”, as if they took place in a remote past. Through the character of Charinus (representing a younger version of himself, although the name is perhaps a play on that of Tschirnhaus), he says: “But when it came to motion, all my care and diligence were in vain... For I always became stuck at the very beginning of an incipient motion, since I had noticed that what must come about in the whole of the remaining time must somehow already happen at the first moment.” (A VI iii N78: 532)

Here we have a subtle allusion to the Inverted Dichotomy establishing endeavours, but that’s all. In the ensuing analysis Leibniz pursues the subject entirely classically, making dextrous use of the traditional paradoxes of continuity known to the ancients, in particular, Plato, Aristotle, the Stoics and Sextus Empiricus. By means of these he establishes there can be no state of change in a moment, and that “whatever changes is in two opposite states at two neighbouring moments.” The model here is that of a line that is actually divided so that its parts are contiguous, not truly continuous. Accordingly, the division of the interval is marked by not one point, but two: the end of the first subinterval, and, immediately next to it, the beginning of the second. There cannot, however, be more than two such neighbouring moments immediately next to each other without the continuum being composed of points, an outcome he rejects as implying a contradiction: “And although the moments and points that are assigned are indeed infinite, there are never more than two immediately next to each other in the same line, since indivisibles are nothing but bounds” (565). This appeals to the conception of actually infinite division of bodies described above, where every portion of matter is “actually divided into further parts, so that there is no body so small that there is not a world of infinitary creatures in it,” but without entailing a division of space or body into points (565). Now Leibniz extends this conception to motion as well: “the motion of a moving thing is actually divided into an infinity of other motions, each different from the other, and does not persist the same and uniform for any stretch of time." (565)

Actually, even more revealing are the marginal notes Leibniz had made for himself on how to improve the first draft of the dialogue:

> (N.B. Just as bodies in space form an unbroken connection, and other smaller bodies are interposed inside them in their turn, so that there is no place void of bodies; so in time, while some things last through a momentaneous leap, others meanwhile undergo more subtle changes at some intermediate time, and others between them in their turn. And in these (as it were) blows or vibrations there seems to be a wonderful harmony. At any rate, it is necessary for states to endure for some time or be void of changes. As the endpoints of bodies, or points of contact, so the changes of states. Smaller bodies move more quickly in a plenum, larger ones more slowly. Nor is any time or place empty. During any state whatsoever some other things are changing.) (A VI iii N78: 559)

This idea of momentaneous leaps is intriguing on two counts. First, the concept 'momentaneous' is given a thoroughly Archimedean (finitist) interpretation: no matter how small a time something endures between changes, something else undergoes subtler changes within this time, thus giving further assignable times within. Second, the leaps in question are not discontinuous leaps from one interval or state to the next, but continuous leaps from the beginning of the interval to its end.[27] But despite their finitude, they are smaller than can be perceived by a body of the same scale: "Matters are to be explained in such a way that bodies never perceive these leaps. So when a large body leaps smaller bodies will also leap, but they need a longer time." (A VI iii N78: 569)

If, however, a body undergoing a leap is undergoing no change, it is not acting, and might as well be said not to exist between the beginning and end of the interval. Leibniz prompts himself to declare this: "Why not rather say that the conclusion that things exist only for a moment, and do not exist at an intermediate time, will follow if it is supposed

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[27] As such they are very reminiscent of the Neoplatonist Damascius, who "gives the name 'leap' not, as we might expect, to the instantaneous transition from one time to another, but to the intervening period between two transitions" (Richard Sorabji, "Introduction", in Simplicius, *Corollaries on Place and Time*, trans. J. O. Urmson: Cornell University Press, Ithaca, 1992.)

that things do not exist unless they act, and do not act unless they change?" (A VI iii N78: 558, marginal note). When he rewrites this section of the dialogue, this conclusion is brought to the fore:

> Pacidius: But I would like you to notice something else, that this demonstrates that bodies do not act while they are in motion.
>
> Theophilus: Why is that?
>
> Pacidius: Because there is no moment of change common to each of two states, and thus no state of change either, but only an aggregate of two states, old and new; and so there is no state of action in a body, that is to say, no moment can be assigned at which it acts. For by moving the body would act, and by acting it would change or be acted upon; but there is no moment of being acted upon, that is, of change or motion, in the body. Thus action in a body cannot be conceived except through a kind of aversion. If you really cut to the quick and inspect every single moment, there is no action. Hence it follows that proper and momentaneous actions belong to those things which by acting do not change. (A VI iii 566)

Here we have a third avatar of the Inverted Dichotomy, even if it is not as explicit as the others.[28] For the argument is that in an apparently continuous motion there is motion, and therefore action, in as small an interval as you wish to choose. But there is no action in a body at any assignable moment: there is only an aggregate of two states, old and new. And between these assignable moments (in what used to be the unassignable intervals) there are arbitrarily small leaps, but which are void of changes, and therefore of action. Consequently the action underlying a continuous motion is not in the moving body. From this Leibniz concludes that bodies' existence requires their creation by God at each assignable moment, though not in between, a doctrine he dubs "transcreationism".

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[28] As Samuel Levey has reminded me, however, there is a more explicit instance of the Inverted Dichotomy in the dialogue. This is when Leibniz transposes the argument of the Sorites from discrete to continuous quantity, concluding that one body must become near to another by the gaining of a part of the line "smaller than any named by us," i.e. a minimum (A VI iii 540). I might well have made more of how it is an extension of this line of argument by a kind of passage to the limit that justifies Leibniz's later conception of monadic states as instantaneous states of change.

Even in his stating of the argument for transcreation, however, Leibniz signals that he has not abandoned the idea of *substances* (in the plural): "those things which by acting do not change," and which have momentaneous actions in the strict sense, are what he will later call "simple substances". Those substances have states which are strictly instantaneous, and are therefore something like the limit of the phenomenal states of the *Pacidius* as the time of their leaps tends to zero. Their action is *appetition*, a tendency or endeavour to change state. This action underpins motion, but does not reside in phenomena, in bodies. But why does Leibniz need a plurality of substances, when the "no-action in bodies" argument supports only a variant of Occasionalism, with God as the one true substance? This is takes us to the subject of my next section, plurality.

But before proceeding, I want to say something more about how these two issues are connected, about how the dynamism underlying motion is linked with the issue of plurality. In the ontology of the *Pacidius* matter is infinitely divided into parts by the differing motions within it, where all parts moving with a motion in common constitute an individual body. The difficulty with this is that a body so individuated does not stay self-identical over time. Any given body may not, of course, be the body of anything that perdures through its changes. But if there were no such perduring elements in matter at all, there would be nothing to prevent the dissolution of a body all the way down into points. Earlier in the year Leibniz had tried to solve this difficulty by positing *atoms* that contained *minds* as their principles of unity. But in the *Pacidius* he rejects atoms ("ôr perfectly solid bodies") in favour of a conception of matter where each body retains a certain tenacity and elasticity, which must be accounted for in terms of the motions of its constituent parts. But how does one account for these motions, for the idea of a body remaining one whilst internally folding and unfolding in multiple creasings? Unless there is some law followed by the motions causing the divisions, internal to the body, there is nothing in it to make it one. But at this point (Fall of 1676) he has not managed to identify a formal principle that would be adequate to such a task. This, I believe, explains his relapse into the Occasionalistic theory he had proposed seven years before.[29] He has established the need for principles of unity, but has no principle of multiplicity.

All this changes with Leibniz's discovery of the conservation of force in early 1678. In a set of metaphysical reflections penned not long afterwards (A VI iv N267; Summer 1678-Winter 1680/81) he notes that the key to a given body's being one despite the multiplicity of its divisions is not the conservation of matter or motion, but that of *power*. "Anyone seeking the primary sources of things," he writes there, "must investigate how matter is divided into parts, and what their motion is"; his own investigations show that

> A unity must always be joined to a multiplicity to the extent that it may. So I say that matter is divided not even into parts equal in bulk, as some have supposed, nor into parts equal in speed, but into parts of equal power, but with bulk and speed unequal in such a way that the speeds are in an inverse ratio to the size" (A VI iv N267: 1401-2).

This, I believe does much to explain why Leibniz's mature theory of substance follows hard on the heels of his discovery of the conservation of force. A corporeal substance has no constant shape or size: it constantly undergoes deformations of shape and size depending on the motions of its components. But it retains the same total quantity of force, which gets differently distributed among its constituent parts from one moment to the next in such a way that quantity of motion is also conserved in all the collisions. This *ratio*[30] for the motions of its constituents is also the reason for its divisions, both of which are infinite, as is the total force of the substance. Clearly much more needs to be said on all this. But this is perhaps enough to establish the

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[29] See Leibniz's Letter to Thomasius of April 30, 1669 (A II i 23-24). I discuss this theory briefly in my review article on Beeley's book on pp. 34-35, *Leibniz Society Review* 7, 1997, 25-42. See also Beeley, *op. cit.* n. 9, 132-133.

[30] For a lucid explanation of Leibniz's concept of *ratio*, as well as of the Platonist underpinnings of Leibniz's early work in general, see Christia Mercer's forthcoming *Leibniz's Metaphysics: Its Origins and Development* (Cambridge: Cambridge University Press, 2001).

embroilment of the question of principles of unity with that of plurality in Leibniz's mature work, to which I now turn. Again let me begin with a comparison to the Eleatics.

Eleatic Arguments Against Plurality

Although there is some dispute as to whether Parmenides considered the One ("what is") to be material, Melissus argued directly for its lack of bulk as follows:

> If it existed, it would have to be one; but if it were one, it could not have body; for if it had bulk, it would have parts; and then it would not be one.[31]

Using this as supplement to the reports of ancient sources one may conjecture a reconstruction of the missing first part of Zeno's argument against plurality as follows:

> If there were many things, each of them would have to possess unity and self-identity. But nothing can have unity if it has size; for whatever has size is divisible into parts, and whatever has parts cannot be one. Hence, if there were many things, none of them could have size.[32]

We see that despite the huge gap in time that separates Leibniz from the Eleatics, he agrees not only with the premise of this reconstruction of Zeno's reasoning, but also with the conclusion—provided "thing" is interpreted in the strict sense as "simple substance". This, incidentally, is no stretch: Leibniz agrees with the Ionians and Eleatics that whatever *is* in the strict sense, is without generation and destruction. Thus in his Reply to Foucher we find him giving the following argument characteristic of his mature thought on substance:

> [S]ince every multiplicity presupposes true unities, it is evident that these unities cannot be matter, otherwise they would in turn be multiplicities, and by no means true and pure unities, such as are finally required to make a multiplicity. Thus the unities are properly substances apart, which are not divisible, nor consequently perishable. (G VII 552)

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[31] Gregory Vlastos, "The Eleatics", p. 24 (I have substituted 'bulk' for his 'thickness'.)

[32] This follows Vlastos in spirit, although I have taken the liberty of reworking his reconstruction of pp. 24-5.

But of course Zeno's conclusion is merely a preliminary one, the first leg of a dialectical argument; he is not arguing, as Leibniz is, for the conclusion that there are many unextended or immaterial unities. In the fragments that have been preserved by Simplicius, he goes on to argue that it is impossible for the extended to be composed of elements each having no size:

> For if it were added to another, it would make it no larger. For having no size, it could not contribute anything by way of size when added. And thus the thing added would be nothing. If indeed when it is subtracted from another the latter is not reduced, nor again increased when it is added, it is clear that what is added or subtracted is nothing.[33]

Having established that the many, if they exist, must each have a (finite) size, Zeno then argues that when added together they will be infinite:

> So if the many exist, each one must have some size and thickness, and one part of it must extend beyond the other. And the same reasoning holds of the projecting part. For this too will have size and some part of *it* will project. Now to say this once is as good as saying it forever. For no such part will be the last, or without one part related to another [in this way].
>
> Thus, if there are many, they must be both small and great; on the one hand, so small as to have no size; and on the other, so large as to be infinite.[34]

On Vlastos' interpretation, this argument again exploits the Dichotomy argument schema: if one takes a part (say a half) of a thing with (finite) size, then a half of the original will be left extending beyond this; if one takes a half of the remainder, a half of that will be left extending beyond it, and so on to infinity. Therefore there will be infinite parts with (finite) size, so that the supposedly finite thing will in fact be infinite. The fallacy here (Vlastos explains) is to suppose that an infinite number of finite parts must add up to an infinite whole. Zeno might have recognized this himself (one can "see" that if one takes a half a square, then half the remainder, and so on to infinity, the result

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[33] Diels, B2; again I owe the translation to Vlastos, "The Eleatics", p. 25. This reasoning is similar to that by which Leibniz (on my construction) comes to reject the infinitely small in 1676.

[34] Vlastos, "The Eleatics", p. 25.

never exceeds the original square). But that does not amount to a logical proof, and no lesser intellects than Galileo and Gassendi were persuaded of the validity of this argument. The first to prove rigorously that a converging infinite series of this sort adds to a finite whole was Gregory of St. Vincent. Again, as we saw above, Leibniz knew of his work, and was powerfully influenced by it. This is what emboldened him to claim that matter can be (and is) divided into an actual infinity of finite parts without there being any smallest part.

Leibniz and Plurality: Russell's Charge

We have seen that Leibniz agrees with the (conjectural) first leg of the Eleatic argument concerning the many unities being unextended, whilst rejecting the second leg on the grounds that infinitely many finite extended reals will not add up to an infinite whole. But this does not yet yield a full and coherent position. In fact many commentators, pre-eminent among them Bertrand Russell, have claimed that Leibniz does not have a consistent position. Leibniz's "whole deduction of Monadism from the difficulties of the continuum," Russell writes,[35] "seems to bear a close analogy to a dialectical argument." By this he does not mean that Leibniz is engaged in Zeno's kind of dialectic, but Hegel's. As he explains, a "dialectical" argument in this pejorative sense is one where "a result is accepted as true because it can be inferred from premisses admittedly false, and inconsistent with each other"; adding sarcastically: "Those who admire these two elements in Hegel's philosophy will think Leibniz's argument the better for containing them." He states Leibniz's argument as follows:

> The general premiss is: Since matter has parts, there are many reals. Now the parts of matter are extended, and owing to infinite divisibility, the parts of the extended are always extended. But since extension means repetition, what is repeated is ultimately not extended. Hence the parts of matter are ultimately not

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[35] Bertrand Russell, *A Critical Exposition of the Philosophy of Leibniz* (1900), p. 110. I gave a fuller analysis of this topic in my "Russell's Conundrum: On the Relation of Leibniz's Monads to the Continuum," pp. 171-201 in *An Intimate Relation*, ed. J. R. Brown and J. Mittelstrass (Dordrecht: Kluwer, 1989).

> extended. Therefore it is self-contradictory to suppose that matter has parts. Hence the many reals are not parts of matter. (The argument is stated almost exactly in this form in G VII 552).[36]

He then remarks: "It is evident that this argument, in obtaining many reals, assumes that these are parts of matter—a premiss which it is compelled to deny in order to show that the reals are not material." Leibniz would have done better, in Russell's eyes, to have gone the whole Eleatic hog: he should have concluded with Spinoza that there is only one substance.[37]

I hope it is evident from the preceding that Leibniz could not have committed the travesty of logic of which Russell accuses him. In none of the avatars of the Inverted Zeno's Dichotomy does Leibniz assume that the parts of what is divided are the reals (i.e. substantial): quite the contrary, in each case it is shown that the extended is inadequate to capture its real beginning or foundation. The extended parts presuppose beginnings or foundations which cannot themselves be extended. In this respect the passage paraphrased by Russell (already quoted in part above) is no exception:

> Everybody agrees that matter has parts, and is consequently a multiplicity of many substances, as would be a flock of sheep. But since every multiplicity presupposes true unities, it is evident that these unities cannot be matter, otherwise they would in turn be multiplicities, and by no means true and pure unities, such as are finally required to make a multiplicity. Thus the unities are properly substances apart, which are not divisible, nor consequently perishable. For whatever is divisible has parts, which can be distinguished even before their separation. (G VII 552)[38]

Here again one sees that Leibniz does not claim that matter's parts *are* the true and pure unities, but that "every multiplicity *presupposes* true unities". To be charitable, one can understand how Russell might have been misled by this wording, where the "parts of matter" referred to in the first leg of the argument are corporeal substances like sheep, whereas the unities that are "properly substances apart" cannot have any distinguishable

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36 *Critical Exposition*, p. 100, n. 1.

37 *Ibid.*, 115ff., 126, 186.

38 This is Russell's own translation, quoted from *Critical Exposition*, p. 248.

parts, and therefore cannot be material. Still, the fact remains that for Leibniz the unities are immaterial things presupposed by any actual part, not the actual parts of matter themselves. This is much clearer in some (unpublished) remarks Leibniz made concerning the same example of a flock of sheep, in drafting a reply to Foucher's objections to his *New System*:

> In realities, where only divisions actually made enter, the whole is only a result or assemblage, like a flock of sheep. It is true that the number of simple substances in any mass, however small, is infinite; for besides the soul, which makes the real unity of the animal, the body of the sheep, for example, is actually divided, i.e. is an assemblage of invisible animals or plants, similarly composite except for what makes their real unity; and though this goes on to infinity, it is plain that all in the end depends on these unities, the rest, or the results, being only well-grounded phenomena. (G IV 492)

For the sake of brevity, this position can be expressed in an appropriately Leibnizian way through a kind of *characteristic*, as follows:

Each whole body B is an aggregate of the parts P into which it is actually divided. Each of these parts is either a corporeal substance C, or an aggregate of corporeal substances. A corporeal substance is a composite of organic body together with an immaterial unifying principle U. The organic body is in turn an aggregate of parts. Thus, using the symbols + for aggregation and $\oplus$ for substantial composition, we have

(1) $B = P + P + P$ (2) $P = C$ or $C + C + C$ and

(3) $C = B \oplus U$

Now (1) and (2) together yield

(4) $B = C + C + C + C + \ldots$ ("body is an aggregate of real unities")

Combining this with (3) gives

(5) $B = (B \oplus U) + (B \oplus U) + (B \oplus U) + (B \oplus U) + \ldots$

This formula is recursive. Evidently (4) and (3) allow substitution for B on the right side *ad libitum*. But after any finite number of substitutions, we will always have a combination of material bodies and immaterial principles of unity. ("I think that no finite substances exist separate from all body"—to de Volder, June 20, 1703; G II 253.) Yet in the final analysis (infinite substitution) we will have

$$(6)\ B = (U \oplus (U \oplus (U \oplus (U \oplus \ldots)))) + (U \oplus (U \oplus (U \oplus (U \oplus \ldots)))) + \ldots$$

("all in the end depends on these unities, the rest, or the results, being only well-grounded phenomena"). Matter does not need to be posited separately by God, but results immediately from his creation of substantial unities.[39]

This *characteristic* displays well, I believe, the essentiality of matter and corporeal substance to Leibniz's argument for immaterial unities, and how their infinite plurality follows as a consequence from the actually infinite division of matter by the Inverted Dichotomy.

To summarize Leibniz's argument for plurality: he accepts the first leg of the Eleatic argument against plurality, which has as a conclusion that what exists (ultimately) must be ungenerable, indestructible and therefore non-material. But he inverts the reasoning in the second. Where Zeno had used the Dichotomy to argue that the parts-within-parts of matter would issue in an infinite whole, Leibniz denies this. But now the Inverted Dichotomy proves that any actual part of matter must have a foundation in reality that is non-material. An actual infinity results: but an actually infinite plurality of reals, not an actually infinite quantity.

Conclusion

This comparison with Zeno has, I hope, thrown some light on Leibniz's doctrine of substance. As is well known, Leibniz holds that matter and motion, taken in themselves, are mere phenomena. Commentators (since Russell, at least) have been prone to interpret this as committing him to a kind of Berkeleian phenomenalism. But this is to misconstrue Leibniz's motives. His intention is not to show the non-existence of matter and motion outside the mind of God, but their insufficiency as currently understood. On

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[39] Here there is an evident analogy with Leibniz's stance on relations, which do not need to be separately posited, but also result immediately when substances are posited. See Massimo Mugnai's *Leibniz' Theory of Relations* (Stuttgart: Franz Steiner, 1992) and his contribution to this volume.

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the standardly accepted accounts, he claims, neither the continuity of motion nor the continued existence of matter are properly intelligible. They must be supplemented by a proper account of substance, which will necessarily be non-geometrical. If matter and motion were unreal, even when well-founded in the substantial actions undergirding them, Leibniz would indeed be guilty of a paralogism in taking them as premises in his arguments for substantial action and plurality. But they are not. As I have argued elsewhere,[40] the reality of the phenomenon of actually divided matter is crucial to Leibniz's argument for substance, not just in his middle period, but throughout his mature work.[41]

The analysis given here extends that argument by giving the parallel case for the phenomenon of motion, and also by showing the necessity of matter to Leibniz's argument for plurality. When the Inverted Dichotomy is applied to motion, the reality of motion requires there to be a foundation of change, a kind of substantial action (*conatus*, active force or appetition) existing as it were between pairs of assignable moments. When it is applied to matter, the reality of matter —secondary matter, actually divided matter— requires there to be a substantial foundation for each real part of matter. This is crucial to the argument for plurality. Without it, the claim that matter presupposes a real unity will not generate more than one real, leaving monism as the only option.

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[40] See my "Russell's Conundrum" of 1989 (op. cit. fn. 35), and "Infinite Aggregates and Phenomenal Wholes," *Leibniz Society Review*, **8**, 25-45, December 1998. See also Pauline Phemister, "Leibniz and the Elements of Compound Bodies," *British Journal for the History of Philosophy* **7**, 1, February 1999, 57-78.

[41] Cf. "If there were no divisions of matter in nature, there would be no diverse things, indeed there would be nothing but the mere possibility of things; but the actual division in masses makes things that appear distinct, and supposes simple substances." (Leibniz to de Volder, 1704-5; G II 256)

Thus Leibniz is not a phenomenalist à la Berkeley.[42] Even though matter and motion are phenomenal and not substantial, the reality of the phenomenon of motion is crucial to the argument for appetition or the continuity of action in each substance; and the reality of the phenomenon of matter is crucial to the argument for an infinite plurality of substances: "*Forma substantialis seu Anima est principium unitatis et durationis, materia vero multitudinis et mutationis*." (A VI iv 1399)

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[42] Here my conclusions are complementary to those of Professor Hans Poser in this volume. The phenomenon of motion presupposes continuous substantial action, and *emanates* from it; and matter emanates from the substantial unities it presupposes. Well-founded phenomena are real in Leibniz's philosophy, even if they are not as real as substances, and these are not as real as God.

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