Excerpts from A New Physical Hypothesis:

Theory of Abstract Motion[1] A VI.ii N41

[Winter 1670/71?]

Predemonstrable Foundations

(1) *There are actually parts in the continuum*, contrary to what the most acute Thomas White believes,[2] and

(2) *these are actually infinite*,[3] for Descartes' "indefinite" is not in the thing, but the thinker.[4]

(3) *There is no minimum in space or body*, that is, nothing which has no magnitude or part.[5] For such a thing has no situation, since whatever is situated somewhere can be touched by several things simultaneously that are not touching each other, and would thus have several faces; nor can a minimum be supposed without it following that the whole has as many minima as the part, which implies a contradiction.[6]

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[1] The full title is *Theory of Abstract Motion, or, The Universal Reasons for Motions, Independent of Sense and the Phenomena*, by the author G.G.L.L. (sc. Gottfried Wilhelm Leibniz of Leipzig), and it is dedicated to "the recently established Illustrious French Royal Society, for the promotion of Mathematics, Physics, Medical Studies, and for increasing the conveniences of the human race." It is the companion piece to the *Theory of Concrete Motion*, which Leibniz sent to the English Royal Society. Together they make up *A New Physical Hypothesis* , whose subtitle is: *by which the causes of most of the Phenomena of Nature are derived from a certain unique universal motion, supposed in our world, without disdaining either the Tychonians or the Copernicans*. I have translated most of the "Predemonstrable Foundations" (264-267), and the beginning of the "Uses" (273), but not the preface or the "Definitions" preceding them (261-264), nor the "Theorems", "General Problems", or "Special Problems" between (268-273).

[2]Thomas White, *Quaestio praevia: Utrum in continuo sunt partes actu* ("A Leading Question: Whether there are actually parts in the continuum"), §§ 1 and 2.

[3] Cf. Axiom 4 from the *Dissertatio de Arte Combinatoria* of 1666: "Every single body has infinite parts, or, as is commonly said, the Continuum is divisible to infinity." (A VI.i 169) The "parts" here might be thought to be merely potential parts, in keeping with the standard Aristotelian analysis (see Appendix 2a), but as Philip Beeley has established, the context makes clear that these parts are conceived by Leibniz as actual parts of the continuum. See Beeley, *Kontinuität und Mechanismus*, pp. 56-57.

[4] This was a common reaction to the distinction between the infinite and the indefinite which Descartes had elaborated in his *Principia philosophiae*, part 1, § 26-27, and applied to the infinite division of matter in part 2, § 34-35 (see Appendix 2c).

[5] Cf. Euclid's definition, "*A point is that which has no part, or has no magnitude*", *Elements*, Book 1, Definition 1.

[6] Presumably Leibniz is here thinking of any of the standard geometrical demonstrations that a shorter line contains the same number of assignable points as a longer one, such as could be found, for instance, in Froidmont's *Labyinthus de compositione continui*. In "On Minimum and

(4) *There are indivisibles or unextended things*, otherwise neither the beginning nor the end of a motion or body is intelligible. This is the demonstration: any space, body, motion and time has a beginning and an end. Let that whose beginning is sought be represented by the line ab, whose midpoint is c, and let the midpoint of ac be d, that of ad be e, and so on. Let the beginning be sought to the left, on a's side. I say that ac is not the beginning, since dc can be taken away from it without destroying the beginning; nor is ad, since ed can be taken away, and so on. Therefore nothing is a beginning from which something on the right can be taken away. But that from which nothing having extension can be taken away is unextended. Therefore the beginning of a body, space, motion, or time (namely, a point, an endeavour, or an instant) is either nothing, which is absurd, or is unextended, which was to be demonstrated.

(5) *A point is not that which has no part*,[7] nor that whose part is not considered;[8] but that which *has no extension*, i.e. whose parts are indistant, whose magnitude is inconsiderable, unassignable, is smaller than can be expressed by a ratio to another sensible magnitude unless the ratio is infinite, smaller than any ratio that can be given. But this is the basis of the *Cavalierian Method*, whereby its truth is evidently demonstrated, inasmuch as one considers certain rudiments, so to speak, or beginnings, of lines and figures smaller than any that can be given.[9]

(6) The ratio of rest to motion is not that of a point to space, but that of nothing to one.

(7) Motion is continuous, i.e. not interrupted by any little intervals of rest.[10] For

(8) once a thing comes to rest, it will always be at rest, unless a new cause of motion occurs.

(9) Conversely, that which is once moved always moves, insofar as it is able,[11] with the same velocity and in the same direction.

Maximum …" (Aiii5) above, Leibniz gives a version of the argument that Galileo Galilei had presented in his *Two New Sciences*.

[7] As noted above, this is Euclid's definition of a point.

[8] This is Hobbes's definition: "*a point is that whose quantity is not considered*"; cf. the definitions from *De Corpore* given in Appendix 2d below. It is defended by him in his *Six Lessons to the Savilian Professors of the Mathematics* as equivalent to Euclid's, which he gives as *Signum est, cujus est pars nulla* (*The English Works of Thomas Hobbes*, vol. VII. London, 1845; reprinted by Scientia Verlag Aalen, Germany, 1966; pp. 200-202).

[9] Leibniz is here referring to Bonaventura Cavalieri's *Geometria indivisibilibus continuorum nova quadam ratione promota*, 1635 and 1653. See GLOSSARY notes on *indivisibilis*, *infinitesimalis*.

[10] This is the Arriagan analysis of motion popularized by Gassendi (see Appendix 2e), which Leibniz himself had subscribed to some years earlier. Here he gives his reason for abandoning it as its incompatibility with (8), the "inertial" principle for rest, subscribed to by Aristotle, Descartes and Gassendi himself. See the discussion in the *Pacidius*, section II above.

[11] Leibniz's Latin here is *quantum in ipso est*, close to the *quantum in se est* of Descartes and Newton. See I. Bernard Cohen, "'Quantum in se est': Newton's Concept of Inertia in Relation to Descartes and Lucretius," *Notes and Records of the Royal Society of London*, 19 (1964): 131-155.

(10) Endeavour is to motion as a point is to space, i.e. as one to infinity, for it is the beginning and end of motion.[12]

(11) Whence, *whatever moves* —however feebly, and however great the obstacle— *will propagate an endeavour through all obstructions in the plenum* to infinity, and will therefore impress its endeavour on everything else.[13] For it cannot be denied that even when it ceases to continue moving, it at least endeavours to do so; and therefore that it endeavours —or what is the same thing, begins— to move any obstacles, however large, even if it is overcome by them.

(12) Thus there can *be several contrary endeavours in the same body simultaneously*. For if there is a line ab, and c tends from a to b, and d, in turn, tends from b to a, and they collide; then at the moment of collision c will endeavour towards b, even though it be thought to have stopped moving, since the end of a motion is an endeavour; but it will also endeavour in the reverse direction, if the opposing body be thought to prevail, for it will begin to move backwards. But it will be all the same even if neither body prevails, since every endeavour is propagated through obstructions to infinity, and thus the endeavour of each will be propagated into each; and if it makes no difference when their speeds are equal, then neither will it when one is double or however much greater than the other, since twice nothing is nothing.

(13) *One point of a moving body in the time of its endeavour,* i.e. in a time smaller than can be given, *is in several* places or *points of space*, that is, it will fill a part of space greater than itself, or greater than it fills when it is at rest, or moving more slowly, or endeavouring in only one direction; yet this part of space is still unassignable, or consists in a point, although the ratio of a point of a body (or of the point it fills when at rest) to the point of space it fills when moving, is as an angle of contact to a rectilinear angle, or as a point to a line.

(14) But in general, too, *whatever moves is never in one place while it moves*, not even at an instant or minimum of time; since that which moves in time, endeavours to move in an instant, that is, it begins and ceases to move, i.e. to change place. And it is no good saying that what endeavours at any time smaller than can be given, is really at a minimum in place: for there is no minimum part of time, otherwise there would also be one of space. For whatever completes a line in a time will, in a smaller time than can be given, complete a smaller line than can be given, that is, a point; and in an absolutely minimum time, will complete an absolutely minimum part of space; but there is no such thing (*by Foundation 3*).

(15) On the other hand, *at the time of collision*, impulse, or impact, the two extrema or endpoints of the colliding bodies mutually penetrate, i.e. *are in the same point of space*: for since one of the colliding bodies endeavours to move into the other's place, it begins to be in it, i.e. it begins to penetrate, or be united. For endeavour is a beginning, penetration is a union: thus they are at the beginning of a union, i.e. their bounds are one.

(16) Therefore *bodies which press against* or impel *one another, cohere*: for their bounds are one, and those things whose extremities are one [ων τα εσχατα εν] are, by Aristotle's definition

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[12] See Hobbes's definition in *De Corpore*, given in Appendix 2d.

[13] Compare with Hobbes, De Corpore, Part II, ch. XV, §7: “All endeavour is propagated to infinity.”

too,[14] continuous, i.e. cohering, since if two things are in one place, one cannot be impelled without the other.[15]

(17) *No endeavour lasts longer than a moment without motion, except in minds*. For that which in a moment is an endeavour, is in time a motion of a body. And here a door is opened for pursuing the true distinction between bodies and minds, till now explained by no one. For every body is a momentaneous mind, i.e. a mind lacking *recollection*, since it does not retain its own endeavour and a contrary one together for longer than a moment. (For two things are necessary for *sensing pleasure* or pain: namely, action and reaction, that is to say, comparison and thus *harmony* —and there is no sensation without these). Therefore body lacks memory, it lacks a sense of its own actions and passions, it lacks thought.

(18) *One point is greater than another, one endeavour is greater than another, but one instant is equal to another*, whence time is expounded by a uniform motion in the same line,[16] although its parts do not cease in an instant, but are indistant. In this they are like the angles at a point, which the Scholastics (whether following Euclid's example, I do not know) called *signs*, as there appear in them things that are simultaneous in time, but not simultaneous by nature, since one is the cause of the other. Likewise in accelerated motion, which, since it increases at any instant, increases at once from the beginning; but to increase presupposes an earlier and a later; so in this case it is necessary for there to be one sign prior to another in a given instant; though without distance or extension (add *problems 24, 25*.)[17] The inequality of endeavours no one will easily deny, but the inequality of points follows from it. It is clear that one endeavour is greater

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[14] See Aristotle, Physics, VI 1, 231a 19; given in Appendix 2a.

[15] That Leibniz was proud of this demonstration is clear from his attempt to urge it on Hobbes himself. In his letter to Hobbes of July 28, 1670, he wrote: "I would have thought that the endeavour of the parts towards one another —i.e. the motion by which one presses upon another— would suffice to bring about the cohesion of bodies. For those things which *press upon* each other are in an endeavour to penetrate. Endeavour is a beginning, penetration is a union. Therefore they are at the beginning of a union. But those things are at the beginning of a union whose beginnings or boundaries are one. Now things whose boundaries are one, or τα εσχατα εν, are, by Aristotle's definition too, not only contiguous but continuous, and truly one body, movable in one motion" (A II.i 57). See also his letter to Oldenburg of 28 September, 1670 (A II.i 63-64), where he gives another lengthy rendition of this account of cohesion.

[16] This Gassendian doctrine has its origin in Galileo's analysis of falling bodies. Since the time of fall is divided into equal parts, it is assumed that when the division is continued indefinitely, the equality still holds even for the infinitely small parts or moments. Uniform motion is thus that in which equal infinitesimals of space are accrued in equal moments; uniform acceleration that in which equal infinitesimals of velocity are accrued in equal moments. See Gassendi, Letters on Motion, *Opera omnia*, III, (Anisson: Lyon, 1658; reprinted by F. Frommann: Stuttgart, 1964), pp. 478ff.; esp. 564b-565a.

[17] Problems 24 and 25 are respectively to *accelerate* and to *retard* a given continuous motion in a given ratio. Leibniz conjectures: "I think this can occur if at different instants of the same *sign* (see *foundation 18*) different endeavours are impressed on the same body... ." But he concludes with the confession that "these last three problems [sc. 23-25] I have not yet adequately weighed or exactly constructed" (A VI.ii 273).

than another, i.e. that a body which moves more quickly than another already covers more space from the beginning: for if it covers just as much space at the beginning, it will always cover just as much, since as a motion begins, so it continues, unless there is some extrinsic cause changing it (*by foundation 9*)... . Therefore the faster covers more space than the slower in a given instant, but in one instant no endeavour can traverse more than a point, or a smaller part of space than can be expounded, otherwise in a time it would traverse an infinite line. Therefore there is one point greater than another. Whence the unassignable arc of a bigger circle is greater than that of a smaller one: and any line whatever, drawn from the center to the circumference, commensurable with the circle, that is, the line by whose rotation the circle is generated, is a perpetually increasing *minimum sector*, but extensionless within. Hence one also solves the difficulties about the two *concentric wheels* rotating on a flat plain,[18] about *the angle of contact*, and as many others, which the most eloquent *Belin* challenged all the philosophers on earth to explain,[19] and from which the *Skeptics* derive the most triumph. An *angle* is the quantity of a point of intersection, i.e. a portion of a circle smaller than can be assigned, i.e. of a *center* —the whole doctrine of angles is about quantities of unextended things. An *arc* smaller than any that can be given is still greater than its chord, although this is also smaller than can be expressed, i.e. consists in a point. But that being so, you will say, an *infinitangular polygon* will not be equal to a circle: I reply, it is not of an equal magnitude, even if it be of an equal extension: for the difference is smaller than can be expressed by any number. Whence from Euclid's definition: *a point is that which has no part*, no error could creep into demonstrations concerning extension, ... provided a part of extension, i.e. a part distant from another part, is understood... .

(19) *If two endeavours occurring at the same time are conservable, they are composed into one, and the motion of each is conserved*, as is clear in the case of a sphere rotating in a straight line on a plane, where the motion of some specified point on its surface is composed through minima, i.e. through endeavours, out of straight and circular motions combined into a Cycloidal one ... . This argument deserves to be treated more diligently by Geometers, so as to make clear which lines will produce which new lines by the combining of their endeavours; and in this way they will perhaps be able to demonstrate many new Geometrical Theorems. ...

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[18] This is the problem of "Aristotle's Wheel", which Galileo used to support his theory of voids in matter, and may have led Gassendi to propose his theory that the slower motion is one interrupted by intervals of rest. See Appendices 2b and e, and notes therein.

[19] The Akademie editors identify Belin as the author of the anonymously appearing books *Les aventures du philosophe inconnu en la recherche et en l'invention de la pierre philosophale*, Paris 1646, and *Apologie du grand oeuvre*, Paris 1659, but they could not locate the challenge referred to by Leibniz.

On Primary Matter[20] A VI.ii N42$_3$

[1670-1671?][21]

Aristotle's primary matter is the same as Descartes's subtle matter. Each is divisible to infinity. Each lacks form and motion in itself, each acquires forms through motion. Each receives its motion from a mind. Each is formed in certain whirls, and Aristotle's vortices have no greater solidity than do Descartes's. Each kind of vortex gets its solidity from motion, since nothing disturbs it, although Descartes did not himself give this as the cause of solidity. Each whirl propagates its action onto another whirl through impressed motion, on account of the continuity of matter. For Aristotle too, no less than Descartes or Hobbes, derives all particular things solely from the motion of the universal whirls. Whence Aristotle imparted intelligences only to the principal whirls, since from the collisions of these whirls the actions of the others follow. Aristotle erred in making the earth the center of the world and of all the gyrations. But this error should be pardoned, since philosophy had not yet been sufficiently equipped with observations.

To these remarks I now add that *primary matter is nothing if it is at rest*. And this is what certain Scholastics said obscurely when they said that primary matter even obtains its existence from form. There is a demonstration of this. For whatever is not sensed is nothing. But that in which there is no variety is not sensed. Similarly: *If all primary matter were to move in one direction, that is, in parallel lines, it would be at rest,* and consequently would be nothing. *Everything is a plenum*, since primary matter and space are the same. Therefore *every motion is circular*, or is composed of circular motions, or at least joins back up with itself. The several circulations will mutually obstruct each other, or act one upon another. *Several circulations will endeavour to unite into one*, that is, all bodies tend towards rest, i.e. annihilation. *If bodies are devoid of mind, it is impossible for motion to have been eternal*.[22] *The conflicting universal circulations give rise to the particular ones, i.e. bodies. Matter is actually divided into infinite parts. There are infinitely many creatures in any body whatever. All bodies cohere to one another. Yet every body separates from every other, although not without resistance. There are no Atoms*, i.e. bodies whose parts never separate. There are two principles by which motion is varied: compositions of endeavours and compositions of {---}[23]

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[20] Only the slip of paper containing the first paragraph (LH IV 1, 4k, leaf 37; cut off to the right of and beneath the margin) has been preserved and catalogued by Bodemann. But Gerhardt had a second manuscript before him, now missing, which he interleaved with the first in the edition he made; re-edited by the Akademie, pp. 279-80.

[21] On the basis of the style of the handwriting, the Akademie editors assign the first paragraph to the second half of Leibniz's stay in Mainz. The second paragraph was probably written somewhat later, but, on the basis of its contents, the editors ascribe it to the same period.

[22] According to Gerhardt, above "for motion to have been eternal", Leibniz had written "it can be diminished without end."

[23] According to Gerhardt, here a word and a couple of lines were unreadable due to destruction of the paper. It is not clear what the second principle is: the composition of motions was generally accepted at this time, but Leibniz analyzes this in terms of the composition of endeavours.