

From *The Metaphysical Foundations of Mathematics* [ca. 1715]

In his Latin course on Mathematics (see the *Acta Eruditorum* of 1714), the distinguished mathematician Christian Wolff has recently touched on certain reflections of mine concerning the analysis of axioms and the mathematical nature of similarity, and has explicated them after his own fashion.<sup>1</sup> Consequently it seemed appropriate to present here, lest they be lost, certain views which I worked out some time ago, from which it will be understood that there is a certain Method of Analysis of greater scope than Mathematics, from which mathematical science borrows all its most beautiful methods.<sup>2</sup> It seems best, therefore, to begin at a rather more profound level:

*If many states of things are assumed to exist, none of which involves its opposite, they are said to exist simultaneously. Thus we deny that the events of last year are simultaneous with those of this year, for they involve opposite states of the same thing.*

*If one of two states which are not simultaneous involves the reason for the other, the former is held to be earlier, the latter to be later. My earlier state involves the reason for the existence of my later state. And since, because of the connection of all things, my earlier state involves the earlier state of the other things as well, it also involves the reason for the later state of those other things, so that my earlier state is in fact earlier than their later state too. And therefore *whatever exists is either simultaneous with, earlier than, or later than some other given existent.**

Time is the order of existence of those things which are not simultaneous. Thus it is the universal order of changes, when the specific kind of change is not taken into account.

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<sup>1</sup> The *Acta Eruditorum* was one of the first scientific journals, and it was in its pages that Leibniz and the Bernoullis first brought the methods of the differential and integral calculus to the attention of the educated public. Christian Wolff was the most eminent philosopher in Germany at that time, and did much to promote Leibniz's philosophy in the universities.

<sup>2</sup> Here Leibniz is alluding to his *Analysis Situs*, or Geometry of Situations, a lifelong project for a generalized geometry which would enable him to prove the axioms of Euclidean Geometry from certain general (or "metaphysical") principles, such as "The whole is greater than its part." He also thought it would enable him to give shorter and more perspicuous proofs of many well-known geometrical theorems than can be achieved by the methods of either Euclidean or Algebraic Geometry.

Duration is the magnitude of time. If the magnitude of time is uniformly and continuously diminished, time disappears into a Moment, whose magnitude is zero.

Space is the order of coexistents, or the order of existence among things which are simultaneous.

In each of these two orders –in time as well as in space– things are judged to be nearer to or farther from each other according as more or fewer are required for the order between them to be understood. Hence those two points are nearer for which the interposed points maximally determined from them produce something simpler. Such a maximally determined interposition is the simplest path from one to the other, i.e. the shortest and at the same time the most uniform path, namely the straight line, which is less interjected<sup>3</sup> between nearer points.

Extension is the magnitude of space. It is wrong to confound extension with what is extended, as is commonly done, and to regard it as a substance.

If the magnitude of space is uniformly and continuously diminished, it disappears into a point whose magnitude is zero.

Situation is a mode of coexistence. Thus it not only involves quantity but quality too.

Quantity or Magnitude is that which can be recognized in things only by their compresence or simultaneous perception. So we cannot recognize what a foot is, or an ell, unless we actually have something as a measure, which can then be applied to others. Nor in fact can a foot be adequately explained by any definition which does not again involve something of this sort. For even if we say that a foot is twelve inches, the same problem arises concerning the inch, and we do not gain any greater insight from this; nor can it be said whether the notion of the inch or the foot is prior by nature, since it is quite arbitrary which of the two we wish to take as basis.

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<sup>3</sup> This is one of Leibniz's technical terms. In a fragment dating from 1671 (!) he defines it as follows: "A *continuum* is a whole between any of whose parts other parts are interjected. Something is *interjected* between two things if the sum of its distances from each of them is the distance apart of the two things."

Quality, on the other hand, is that which can be recognized in things when they are observed individually, and compresence is not required. Such are the attributes which are explicated by definition or by the various modifications they involve.

Equals are things having the same quantity. Similars are things having the same quality. Hence if two similars are different, they cannot be distinguished except by compresence.

Hence it is evident, for example, that two equiangular triangles have proportional sides, or vice versa. For when the sides are proportional, the triangles are at least similar, since they are determined in a similar way. Moreover, in every triangle the sum of the angles is the same, since it is equal to two right angles; therefore it is necessary for the ratio of the corresponding angles to the sum to be the same in one as it is in the other; otherwise one triangle could be distinguished from the other by this very fact, that is, by being viewed by itself or individually. In this way it is easy to demonstrate things which it previously took many circuitous detours to do.

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A path is the continuous and successive locus of a movable thing.

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There is, moreover, a certain order in the transition of our perceptions when it passes from one to another through intervening ones. This order too we can call a *path*. But since this order can vary in infinite ways, there must necessarily be one simplest order, which would in fact be that order which proceeds according to the thing's own nature through determinate intermediate perceptions, i.e. through those which are related as simply as possible to the two extrema. ... And this simplest order is the shortest path from one to the other, whose magnitude is called *distance*. (GM VII 25)