Dans les corps il n’y a point de figure parfaite:  
Leibniz on Time, Change, and Corporeal Substance  
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I.
The Cartesian theory of corporeal substance is grounded in the simple and beautiful idea that body is to be understood completely in terms of extension—that is, that the attribute of extension constitutes the whole nature or essence of corporeal substance. Descartes advertises this thesis in a famous passage from the *Principia Philosophiae*:  

Each substance has one principal property that constitutes its nature and essence, and to which all its other properties are referred. Thus extension in length, breadth, and depth constitutes the nature of corporeal substance... Everything else that can be attributed to body presupposes extension, and is merely a mode of an extended thing.  

*Principles* i. 53; *AT* viiiia. 25)

The modes of the extended thing that Descartes goes on to mention are *shape* and *motion*, which he says are ‘unintelligible’ apart

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1 This ‘geometrical’ conception of body is evident through many of Descartes’s earlier writings, as well, for instance, in Rule 12 of *Rules for the Direction of the Mind* (c.1628), where ‘shape, extension and motion’ are already being promoted as the ‘simple natures’ essential to body (cf. *AT* x. 419), and Rule 14, where Descartes, after declaring ‘we should be delighted to come upon a reader favorably disposed towards arithmetic and geometry’, moves immediately to a discussion of the geometrical nature of extension in which ‘body’ and ‘that which is extended’ are rapidly identified with ‘extension’ itself (cf. *AT* x. 441 ff.). For a good discussion of Descartes on the nature of body, see Daniel Garber, *Descartes’ Metaphysical Physics* (Chicago: University of Chicago Press, 1992), ch. 3, esp. 63–9, 75–93. See also Margaret Wilson, *Descartes* (London: Routledge, 1978), 83–8, 166–9.

2 Translations of Descartes follow *CSM* 1; translations of Leibniz follow AG, L, and, especially, Richard Arthur’s translation volume, *LOC*. I have sometimes made modifications without comment. Inaccuracies are my own.
from extension. Extension itself is to be 'distinctly recognized as constituting the nature' of body, and indeed of corporeal substance, for 'in this way we will have a very clear and distinct understanding' of body (Principles i. 63; AT viiia. 30–1). Thus the centerpiece of the Cartesian metaphysics of body is the idea 'that the nature of corporeal substance consists simply in its being something extended' (Principles ii. 19; AT viiia. 51). The underlying hope is, of course, that, with the principal component of the theory of corporeal substance simply being the theory of extension, our understanding of the nature of body will finally be guided by the concepts arising in the most perfectly conceived and perfectly understood discipline in all of human knowledge—namely, the science of geometry.

Critics including Leibniz have been quick to point out that this 'geometrical' conception of body, for all its clarity and precision, cannot deliver all the concepts that will be demanded by a correct physics of bodies (notable among them, inertia and antitypy) and, therefore, that the Cartesian theory of corporeal substance is at best incomplete as a total account. But it is Leibniz alone who criticizes the Cartesian account for relying on concepts that are not clearly understood and that in themselves cannot be true of the nature

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To pick a few famous examples: Locke on what he calls 'solidity': 'Solidity is so inseparable an idea from body that upon that depends its filling of space, its contact, impulse and communication of motion upon impulse,' where, by contrast, 'extension includes no solidity, nor resistance to the motion of body, as body does' (Essay II. xiii. 11, 12); and Leibniz on antitypy, speaking here through the character Philaret in his 1711 dialogue 'Conversation of Philaret and Ariste': 'Philosophers who are not Cartesians do not agree that extension is sufficient to form body; they also require something else that the ancients call antitypy, that is, that which renders a body impenetrable to another' (GP vi. 580); Leibniz on inertia: 'I discovered that this, so to speak, inertia of bodies cannot be deduced from the initially assumed notion of matter and motion, where matter is understood as that which is extended or fills space, and motion is understood as change of space or place' (A VI. iv. 1980). And, finally, Leibniz in the Discours de Metaphysique, article 18: 'Force is something different from size, shape, and motion and it can be judged from that that not all that is conceived in body consists uniquely in extension and its modifications, as our moderns are persuaded' (A VI. iv. 1559). For some recent discussions, see Garber, 'Leibniz and the Foundations of Physics: The Middle Years', in Kathleen Okruhlik and James Robert Brown (eds.), The Natural Philosophy of Leibniz (Dordrecht: D. Reidel, 1985), 27–130, at 30, 78 ff.; Robert Sleigh, Leibniz and Arnauld: A Commentary on their Correspondence [Leibniz and Arnauld] (New Haven: Yale University Press, 1990), 166 ff.; Bernard Williams, Descartes: The Project of Pure Enquiry (London: Penguin Books, 1978), 228 ff.; and R. S. Woolhouse, Descartes, Spinoza, Leibniz: The Concept of Substance in Seventeenth Century Metaphysics (London: Routledge, 1993), 94–6, 102–15.
of corporeal substance; indeed, as he puts it in Article 12 of the *Discours de métaphysique* (1686), ‘the notions involved in extension contain something imaginary and cannot constitute the substance of a body’ (A VI. iv. 1545). What was supposed to be the irreproachable geometry of extension in fact presents an inherently flawed basis for understanding the nature of actual bodies; its fundamental concepts are not so transparent after all. This bold strand of Leibniz’s critique appears in several texts from the 1680s in which he advances various forms of an argument concerning bodily shape to reach the conclusion that the Cartesian attribute of extension is not something that applies to actual bodies as they are absolutely or in themselves, but rather it is only something ‘imaginary’—that is, something that is not in things outside of us, but only appears to be (cf. A VI. iv. 70). Consider two such passages from this period, the first from a piece written in 1683 and titled ‘Mira de natura substantia corporea’ (‘Wonders Concerning the Nature of Corporeal Substance’):

Even though extension and motion are more distinctly understood than other qualities, since all the rest have to be explained using them, it must in fact still be acknowledged that neither extension nor motion can be understood distinctly by us at all. This is because, on the one hand, we are always embroiled in the difficulties concerning the composition of the continuum and the infinite, and, on the other, because there are in fact no precise shapes [*certae figurae*] in the nature of things, and consequently no precise motions [*certi motus*]. And just as color and sound are phenomena, rather than true attributes of things containing a certain absolute nature without relation to us, so too are extension and motion. (A VI. iv. 1465)

The second is from the 1686 piece ‘Specimen inventorum de admirandis naturae generalis arcanis’ (‘A Specimen of Discoveries of the Admirable Secrets of Nature in General’):

Indeed, even though this may seem paradoxical, it must be realized that the notion of extension is not as transparent as is commonly believed. For from the fact that no body is so very small that it is not actually divided into parts excited by different motions, it follows that no determinate shape can be assigned [*assignari*] to any body, nor is an exactly [*exactum*] straight line, or circle, or any other assignable shape of any body, found in the nature of things . . . Thus shape involves something imaginary, and no other sword
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can sever the knots that we tie for ourselves by a poor understanding of the composition of the continuum. (A VI. iv. 1622)

If this argument concerning extension, shape, and the continuum has attracted only sparse attention over the years from Leibniz's commentators,⁴ its basic points are nonetheless familiar from many of his writings.⁵ To offer just the simplest outline, Leibniz's argument is cumulatively established across these three points, each being used as a platform to reach the next:

(I) There are no precise shapes in actual bodies, because of the subdivision of the continuum to infinity.

(II) The notions of shape, motion, and extension are not in things outside us but rather are only imaginary, like those of color, heat, and sound.

(III) Shape, motion, and extension are not qualities that can constitute substance.

As is evident from the passages above, Leibniz will typically add as well that extension and motion are not as distinctly understood as is commonly believed and suggest that all this arises from 'a poor understanding' of difficulties of the composition of the continuum. Under the Cartesian attribute of extension, Leibniz seems to say, corporeal substance can neither be nor be distinctly conceived. And what he often does say is that, if there were nothing but motion and extension in them, bodies would not be substances but only phenomena 'like rainbows and mock suns' (A VI. iv. 1648; cf. A VI. iv. 1464–5). Some of Leibniz's most subtle philosophy is involved in assembling the reasoning that leads from (I) to (II) to (III) in this line of argument about precise shapes and imaginary notions; as a whole and in each of its steps it merits close study. But the focus of the present chapter will fall only on its first point, Proposition (I), that there are no


⁵ Cf. A VI. iv. 1545, 1622, 1648; GP ii. 77, 98–9.
precise shapes in things, because of the subdivision of the continuum to infinity, and in particular on just one of Leibniz’s arguments for it.

2.

So, now: why, and in what sense, are there no precise shapes in bodies? And what has this to do with the subdivision of the continuum to infinity?

Leibniz has two central arguments for the claim that there are no precise shapes in bodies, and, as advertised, each rests on considerations about the infinite subdivision of the continuum. One of those two is really the primary argument, showing up in all of the main texts, and it concerns the division of bodies into parts. By contrast the other is only a secondary argument; it concerns the structure of time and change and occurs just once, in a short and somewhat puzzling text from 1686, ‘Dans les corps il n’y a point de figure parfaite’ or ‘There is No Perfect Shape at all in Bodies.’ It is principally this document (hereafter: ‘No Perfect Shape’) and the secondary argument it contains, concerning time and change, that will be under scrutiny in what follows, though there will be several occasions on which we shall appeal to a significant earlier writing that also addresses the nature of time and change in order to flesh out the argument of ‘No Perfect Shape.’ And to begin the discussion, a sketch of the primary argument concerning the division of bodies into parts will first be in order.

In the primary argument for Proposition (1), Leibniz typically contends that, since the universe of matter is in essence ‘a vessel filled [pleno] with liquid’, the motions of any body will be ‘propagated’ across the world to make at least some impact on every other body (A VI. iv. 1646—7). And thus every body is constantly buffeted by an infinity of impulses transmitted through the plenum and raining down on it from all sides. The result of this ongoing exposure to all the motions in the universe is that ‘there is no body so small that it is not actually subdivided’ (A VI. iv. 1647), and indeed every particular body will be actually subdivided into an infinity of distinct parts. But no particular

6 The argument evidently supposes that bodies cannot undergo elastic deformation, in the sense that they cannot change shape without being actually divided into distinct parts; this is a steady underlying feature of Descartes’s own view of bodies (cf. Principles ii. 34—5,
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geometrical shape can exactly describe the infinitely complex actual structure of such infinitely divided bodies, 'for none can be appropriate [satisfacere] for an infinity of impressions' (A VI. iv. 1648). The shapes embedded in traditional geometry—even in its relatively sophisticated early modern form studied and in part developed by Descartes and Leibniz (inter alios)\(^7\)—can offer only approximate and necessarily inexact or imprecise representations of actual bodies.\(^8\) That is the sense in which 'there are no precise shapes in bodies': it is not that bodies are themselves indeterminate or imprecise with respect to shape, but rather that geometrical shapes are imprecise with respect to actual bodies.

'No Perfect Shape' opens with a version of this argument. Leibniz considers, as a simplified case of a body with a precise geometrical shape, a straight line, and he argues that no such thing actually exists in nature:

There is no precise and fixed [arrestée] shape in bodies because of the actual division of its parts to infinity.

AT viiia. 59–60) and one that Leibniz appears to accept. See my 'Leibniz on Mathematics and the Actually Infinite Division of Matter' ['Leibniz on Mathematics'], Philosophical Review, 107 (1998), 49–96, at 51–2.


\(^8\) As will be suggested below, the account of those Leibnizian actual bodies appears to describe them as fractal in structure, and, if so, then it is in fact true that they cannot be exactly described in terms of traditional geometrical shapes. See my 'The Interval of Motion in Leibniz's Pacidius Philalethi [Interval of Motion]' Nous 37 (2003), 371–416. There are further strands in Leibniz's thought about why precise shapes are imaginary that involve the activity of the imagination in (1) plastering over non-uniformities in things that fall below the threshold of conscious perception (and are detected only by 'petite perceptions') to produce the appearance of uniform precise shapes in nature, and in (2) imagining 'fictive' or ideal limit entities of mathematical constructions, such as imagining a circle as a polygon with infinitely many sides—whose sides 'appear only indistinctly' and then, by a further extrapolation, disappear altogether to form a shape with no sides at all (e.g., Leibniz writes: 'For entities of this kind, i.e., polygons whose sides do not appear distinctly, are made apparent to us by the imagination, whence there arises in us afterwards the suspicion of an entity having no sides. But what if that image does not represent any polygons at all? Then the image presented to the mind is a perfect circle', A VI. iii. 499). A close examination of Leibniz's nuanced attitudes toward the limits of mathematical constructions would be demanded by a full account. My thanks to Richard Arthur for his very helpful suggestions on this point at the Florence Leibniz Workshop in November 2000 and in private correspondence.
Let there be, for example, a straight line ABC. I say that it is not exact [qu'elle n'est pas exacte]. For with each part of the universe sympathizing with all the others, it is necessarily the case that, if the point A tends along the straight line AB, the point B should have a tendency in another direction. For with each part A striving to carry with it all the others, but particularly the nearest B, the direction of B will be composed of that of A, together with some others; and it is not at all possible that B, which is indefinitely close to A, should be exposed to the whole universe in precisely the same way as A, in such a way that AB composes one whole that has no subdivision. (A VI. iv. 1613)

There is a little difficulty with this passage, as Leibniz seems to shift between talking about A and B as ‘points’ in the line and talking of them as ‘parts’ of the line (not an equivalence that can be safely assumed and moreover one that his official theory, early and late, explicitly denies: points are never to be parts of lines but only bounds of them; cf. A VI. iv. 1648; GP iv. 491; A VI. vi. 152; GP ii. 370, 520, etc.). But the gist of the argument is clear enough. Since the different parts of any line will be exposed differently to the motions of the whole universe, no line, however small, could be uniform and undivided, as the ‘exact’ straight lines of traditional geometry are imagined to be. Actual lines—such as the edges of bodies or the paths carved out by the vertices of moving objects—must instead be divided into distinct parts that in fact deviate from one another in their directions, however subtly, and thus at best can only appear to form a single exactly straight line (cf. A VI. ii. 255). Generalizing this result from lines to bodies, we find that no actual body could be uniform and undivided in any part, even if bodies might sometimes offer the appearance of being undivided wholes. The constant flux of the finer sub-parts in fact always lays down an infinite subdivision of the whole body, and, once again, this makes the assignment of a ‘precise and fixed shape’ impossible.

At this point in ‘No Perfect Shape’ Leibniz anticipates a rejoinder to this first line of reasoning. (And what we are calling Leibniz’s secondary argument against the existence of precise shapes in things gradually comes into play, as discussion turns to the topics of time and change.) The rejoinder itself is left unstated, but it apparently goes something like this. Suppose a given line or body is taken as it is just in a single
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instant, leaving aside the changes it undergoes in the surrounding interval, and focus only on its momentary shape. This momentary shape might be precise. For if one disregards the differing tendencies toward motion in the distinct parts and considers the shape of the line or the body with its parts ‘fixed’ or ‘frozen’ intact at a single moment, the subdivisions that are about to give way in later moments might not present an obstacle to a precise geometrical description of it at this one instant. Perhaps the line is exactly straight if taken in a single and, so to speak, ‘frozen’ moment.

In a few rather compressed remarks Leibniz offers a reply, or perhaps two replies, to this ‘frozen-moment’ suggestion. This is the first reply to the claim that a momentary precise shape might be assigned to the body in a frozen moment:

It is true that one could always draw an imaginary line each instant, but this line will endure in the same parts only for this instant, because each part has a motion different from every other, since it expresses the whole universe in a different way. Thus there is no body that has any shape for a definite time, however small it might be. Now I believe that what is only in a moment [n’est que dans un moment] has no existence [existence], since it begins and ends at the same time. (A VI. iv. 1613)

If the line really is going to be held exactly straight for some time, it can be so only for an instant: stopping the course of changes in the line so that a single precise and fixed shape might be assigned to it narrows the window of time in which that shape can be assigned down to a single moment. But now, since the allegedly precise shape can exist only in a moment, we find that there is no such shape after all. For, as Leibniz says, ‘what is only in a moment has no existence’.

Yet Leibniz’s defense for the crucial premise of this argument is rather hard to make out, despite its simple statement. Why should it be that ‘what is only in a moment has no existence’? It is perhaps being assumed here that whatever ‘has existence’ must have both a beginning and an end, and that these must be opposite states of it. If so, the hypothesis of something that ‘is only in a moment’ and thus ‘begins and ends at the same time’ would result in contradiction and is thereby reduced to absurdity with little trouble.
Maybe such an easy reductio is what Leibniz has in mind there,9 but we shall not pursue it.10 For in the very next sentence Leibniz’s discussion appears to turn toward a quite distinct reply to the possibility of a precise momentary shape. And this now cuts to the heart of the secondary argument for Proposition (I)—the argument that concerns the nature of time and change. Leibniz writes: ‘I have proved elsewhere that there is no middle moment, or moment of change, but only the last moment of the preceding state and the first moment of the following state. But this supposes an enduring state’ (A VI. iv. 1613).

The three points being made in that passage should be clearly distinguished, since they will have to be considered separately in a short while.

(1) There is no middle moment or moment of change.
(2) Change in fact involves only the last moment of the preceding state and the first moment of the following state.
(3) This analysis of change supposes an enduring state.

When Leibniz says that he has proved all this ‘elsewhere’, he is referring to his 1676 dialogue Pacidius Philalethi, in which he presents his first philosophy of motion and takes on the paradoxes of the composition of the continuum.11 By looking back to the Pacidius we shall be able to understand the grounds for (1), (2) and (3), and this will

9 We should not say that this is definitely not what Leibniz has in mind. In a text that the Akademie editors date to the winter of 1682–83, ‘An Corpora Sint Mera Phaenomena’, Leibniz offers a similar line of argument concerning the composition of a cube from two touching triangles. If the triangles were to touch only for a single moment, he argues, this would not be enough for a cube to exist, for ‘if they are in that situation of the cube only for a moment . . . it would follow that the cube would arise and perish simultaneously’ (A VI. iv. 1464–5). Here ‘arise’ [nasci] and ‘perish’ [mierire] do appear to be opposite states; and there is nothing comparable in that text to the discussion of middle moments that follows in the present one. So perhaps Leibniz has two lines of argument: the too-brief reductio, and the more subtle one explored next.

10 For some discussion of this passage, and of the general line of argument in ‘No Perfect Shape’, see Adams, Leibniz, 231–2. Adams describes the premise that what is only in a moment has no existence as ‘a large and dubious assumption’. See also Sleigh, Leibniz and Arnauld, 211. Marc Bobro has also offered an instructive discussion of that premise in his paper ‘Leibniz on Instantaneous Perceptions: A Momentary Lapse of Reason?’, delivered at the Eastern Division meeting of the American Philosophical Association on December 30, 1999, in Boston.

11 In his ‘Introduction’ to LOC, Arthur also notes this connection between the argument of ‘No Perfect Shape’ and the Pacidius.
throw the entire secondary argument for Proposition (I) into much sharper relief.

3.

Early on in the *Pacidius*, Leibniz offers a detailed discussion of the nature of change (see especially A VI. iii. 535–41), and it is there that he gives his proof that there is no middle moment in change. In this discussion Leibniz handles a few different examples of change, the most vivid being that of death—or what the interlocutors call the ‘act of dying’. The dialogue takes up the question whether the act of dying could occur in a single moment of change that transacts the passage from life into death, where life and death are held to be opposite states. Leibniz’s argument against middle moments is straightforward, and in light of it one might say that it is the ‘logic of change’ itself that precludes middle moments. The argument runs across many lines of the dialogue and culminates in this exchange between Pacidius, who plays the role of Socrates, and Charinus, the principal interlocutor:

PACIDIUS. Isn’t death a change?
CHARINUS. Undoubtedly.
PACIDIUS. I understand it to be the act of dying.
CHARINUS. And so do I.
PACIDIUS. Is someone who is dying alive?
CHARINUS. That’s a puzzling question.
PACIDIUS. Or is someone who is dying dead?
CHARINUS. This I see to be impossible. For to be dead means for one’s death to be past.
PACIDIUS. If death is past for the dead, then it will be in the future for the living, just as someone who is being born is neither about to be born nor already born.
CHARINUS. So it seems.
PACIDIUS. Therefore it is not the case that someone who is dying is alive.
CHARINUS. Agreed.
PACIDIUS. So someone who is dying is neither dead nor alive.
CHARINUS. I concede this.
PACIDIUS. But you seem to have conceded something absurd.
CHARINUS. I do not see the absurdity yet.
PACIDIUS. Doesn’t life consist in some particular state?
CHARINUS. Undoubtedly.
This state either exists or it doesn’t exist.

There is no third alternative.

We say that anything in which this state does not exist is lacking life.

Yes.

Isn’t the moment of death the moment at which someone begins to lack life?

Why not?

Or is it the moment at which he ceases to have life?

Precisely.

I am asking whether life is absent or present at that moment.

I see the difficulty, for there is no reason why I should say one rather than the other.

Therefore you must either say neither or both.

But you have blocked this way out for me. For I see well enough that a given state is necessarily present or absent, and cannot at the same time be both present and absent, nor neither present nor absent.

What, then?

Yes, what?, otherwise I am stuck. (A. VI. iii. 355)

Charinus finds himself stuck, for the hypothesis that ‘the act of dying’ occurs in a single middle moment results in a trilemma. If the act of dying itself were to occur in a middle moment, either (i) it would have to be a combination of two opposite states, life and death, which is contradictory, or (ii) it would have to be a state that is one neither of life nor of death, which violates the principle of the excluded third (Leibniz writes: Tertium nullo est), or, finally, (iii) that middle moment would have to be either the last moment of life or the first moment of death, but there could no reason why it should be one rather than the other.

In the face of this perplexity, the interlocutors conclude that there is no momentary state of change or middle moment. The logic of change, so to speak, rules it out. The occurrence of change itself must therefore instead always take place over some stretch of time.

We concluded that a state of change is impossible.

Yes, we did, if the moment of change is assumed to be the moment of a mediate or common state.

But don’t things change?

Who would deny it?
PACIDIUS. Then change is something.
CHARINUS. Of course.
PACIDIUS. Something other than what we have shown to be impossible, namely a momentaneous state.
CHARINUS. Yes.
PACIDIUS. Then does a state of change require some stretch of time?
CHARINUS. So it would seem. (A VI. iii. 538)

The reality of change, it would seem, is not open for negotiation, even in the face of the present difficulties. If that is a small concession to common sense ("Who would deny it"?), still, it is one that continues to pose a challenge to Leibniz’s inquiry. What could change be? The interlocutors turn their attention to the nature of change and in particular to this ‘stretch of time’ that is required by a state of change. In the first major step, Charinus proposes a new theory of change that identifies change itself as an aggregate of two moments:

CHARINUS. I think I have finally found a way out. For I would say that it [change] is composed from both [the last moment of the earlier state and the first moment of the later state], and that, although it is usually called momentaneous, it in fact contains two moments . . .
PACIDIUS. You have spoken correctly, and consistently with what you said above, so that I have no objection to this opinion of yours. (A VI. iii. 541)

Thus, on the analysis of change that emerges in the Pacidius, a state of change actually consists of an aggregate of two immediately neighboring moments containing two opposite states—'the last moment of the earlier state and the first moment of the later state'. For instance, the act of dying will be an aggregate that consists of the last moment of life and the first moment of death. And thus change minimally requires a 'stretch of time' that includes two moments rather than just one.

It is by this point clear that the discussion of change in the Pacidius provides the account Leibniz draws upon in 'No Perfect Shape' when, in the latter, he reports that there is 'no middle moment, but only the last moment of the preceding state and the first moment of the following state' (A VI. iv. 1613). The logic of change implies (1) that there is no middle moment or moment of change. The new theory of change implies (2) that change is an aggregate of two neighboring moments containing two opposite states. Notice, though, that those facts about the logic of change and the new theory of change do not yet entail
Leibniz's third claim from 'No Perfect Shape'—namely, (3) that this analysis of change 'supposes an enduring state'. For those two facts about change still perfectly well allow that change might be an aggregate of nothing more than two moments containing two opposite momentary states. No enduring state has yet been supposed. To see why the analysis of change additionally requires enduring states—and thus to look still more deeply into the nature of the 'stretch of time' required for change to occur—we need to turn from the discussion of change to the metaphysics of time, and in particular to a strand of thought about the nature of the continuum, once again in *Pacidius Philalethi*.

According to Leibniz, the continuum of time is divided up into intervals, and at each point of division there are moments assigned into the continuum (A VI. iii. 552–3); in fact, since every interval is itself infinitely subdivided into smaller intervals, every interval contains an infinity of moments (A VI. iii. 564–5). But the metaphysical nature of the relation of the individual moments to the continuum of time is of paramount importance. In particular, it is crucial to Leibniz's account of the continuum that moments always be either ends or beginnings of extended intervals of time—the *termini* of intervals—and not independent elements of time. For, if moments are allowed to be independent elements of time, then the continuum of time itself will be an aggregate of nothing but moments, finally dissolving into individual moments as if it were just so many grains of sand. And this will raise the paradoxes of the composition of the continuum.

**CHARINUS.** Supposing we concede to you that space is an aggregate of nothing but points and time an aggregate of nothing but moments, what do you fear so much from this?

**PACIDIIUS.** If you admit this, you will be swamped by the whole stream of difficulties that stem from the composition of the continuum, and that are dignified by the famous name of the labyrinth.

**CHARINUS.** This preface is capable of striking terror into someone even from afar! (A VI. iv. 548)

That is where the trouble really starts, of course, though it is beyond the scope of this chapter to pursue Leibniz's inquiry into that labyrinth.12

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12 For some discussion of that inquiry, see Arthur, 'Russell's Conundrum: On the Relation of Leibniz's Monads to the Continuum', in James Robert Brown and Jürgen
Let us instead move straight to its conclusion. The Pacidius' final resolution of the difficulties of the composition of the continuum, as applied to time, requires that time be infinitely subdivided into intervals, but that there be no moments in time apart from the beginnings and ends of those intervals. This is how Leibniz describes the division of the continuum (and this description covers space, time, motion, and matter).

Accordingly the division of the continuum must be considered to be not like the division of sand into grains, but like that of a sheet of paper or tunic into folds. And so, although there occur some folds smaller than others infinite in number, a body is never thereby dissolved into points or [seu] minima . . . It is just as if we suppose a tunic to be scored with folds multiplied to infinity in such a way that there is no fold so small that it is not subdivided by a new fold . . . And the tunic cannot be said to be resolved all the way down into points; instead, although some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain mere extrema. (A VI. iii. 555)

Just as the tunic is divided into folds within folds ad infinitum, but is not thereby resolved into a mere collection of points, the continuum of time is divided into intervals within intervals ad infinitum, but it is never resolved into a dust of single moments. Moments always remain mere extrema and are never free-standing independent elements of time.

In the face of the fact that independent moments are impossible, and that there is 'only the last moment of the preceding state and the first moment of the following state', the only coherent idea of a state that exists at a moment is the idea of the 'last moment' or 'first moment' of some enduring state. Moments are themselves always ends or beginnings of intervals. Likewise, momentary states are always ends or beginnings of enduring states. This is why in 'No Perfect Shape' Leibniz says, of his own analysis of change, that it 'supposes an enduring state' (A VI. iv. 1613); in fact, since it holds change to be an aggregate of two moments containing opposite states, that analysis

supposes *two* enduring states and *two* extended intervals of time, the one preceding and the one following.

So the logic of change precludes change from taking place in middle moments; the new theory of change demands that change is an aggregate of two moments containing two opposite states; and now the metaphysics of time demands that those two moments containing two opposite states are themselves always end moments of intervals of time and that they contain the momentary ends of enduring states.

With this total account of time and change in hand, we can at last see exactly why the 'frozen-moment' suggestion for assigning a precise shape to a body at a moment goes wrong. The strategy behind the 'frozen-moment' suggestion was to find a stable point in the flux of changing parts at which a shape could be assigned to a body—a 'precise and fixed shape'. The point of singling out a given moment in time was intended to hold the shape of the body 'fixed' (arrestée). The point of considering the body as having a shape that exists only in the moment, and thus of disregarding the tendencies of its parts to move in different directions, was to eliminate from view the subdivisions that will break open in following moments, thereby 'freezing' the parts intact so that the whole body might prove more uniform and tractable to being exactly described by a precise shape of traditional geometry.

The strategy for assigning a precise shape to a body at a moment that imagines a frozen moment in time runs afoul both of the 'logical' and of the metaphysical requirements on change, for it has to suppose that a precise shape will be a non-enduring momentary state that exists in an independent middle moment. But there can be no frozen moment of this sort, since all moments are essentially beginnings or ends of intervals in which motion occurs. And there is no momentary state in things apart from the end of one enduring state and the beginning of the next. Momentary states presuppose enduring states, just as moments themselves presuppose intervals of time. The task of assigning a shape to a body at a moment therefore runs back into the original difficulty of assigning a shape to a moving body, where this shape is never a merely momentary state of it but will always be the beginning or end of an enduring state. If there are to be precise shapes in things, they must be enduring states of them.

This result now leads into a profound consequence of Leibniz's account, one that significantly raises the philosophical stakes in his
critique of the Cartesian theory of corporeal substance. For falling under Leibniz’s attack will be not only the Cartesian notions of shape and extension but also the idea of an enduring state.

4.

Once it has been announced in ‘No Perfect Shape’ that Leibniz’s analysis of time and change ‘supposes an enduring state’, the topic of imprecision comes into play. Leibniz writes:

Now all enduring states are vague \([vagues]\), and there is nothing precise about them. For example, one can say that a body will not leave some place greater than itself for a definite [certaine] time, but there is no place where it may endure that is precise or equal to the body. One can thus conclude that there is no moving thing of a definite shape. (A VI. iv. 1613–14)

Leibniz’s conclusion that ‘there is no moving thing of a definite shape’ can be achieved in more than one way from the premise that enduring states are not precise. To see just what argument is being advanced, that premise itself needs to be examined. Why is it that ‘all enduring states are vague, and there is nothing precise about them’?

Leibniz explicitly calls enduring states ‘vague’, and, given his expertise with the Sorites,\(^\text{13}\) it may well be that he means to claim that the very idea of an enduring state is subject to this classical paradox. Yet it is difficult to see how that charge of vagueness could be substantiated in this case. What is it about an enduring state demands that it be vague? Leibniz’s own example does little to help. If there is no place where a body may endure that is ‘precise or equal to the body’, the problem seems to be that bodies and the places that they occupy are imprecisely fitted to one another. If ‘place’ is taken to be a Cartesian notion, then perhaps a place cannot be ‘precise and equal to the body’ for the same reason as before—namely, that Leibnizian bodies cannot be exactly described by the traditional geometry assumed by the Cartesian account. Even if this is not the reason, however, the idea of an imprecision in fit between place and body seems to rely on the idea that there is something imprecise about the notion of an

\(^{13}\) Leibniz develops perhaps the most sophisticated discussions and uses of the Sorites in early modern philosophy; cf. A VI. iii. 539 ff., A VI. iv. 69–70, A VI. vi. 302, 321.
enduring body, presumably an imprecision concerning shape. What else could it be? But if that is the idea here—if the sub-argument for the vagueness of enduring states presupposes that bodies are imprecise in shape—then the overall argument of 'No Perfect Shape' moves in a circle, since the imprecision of enduring states is actually and explicitly being used in that piece as a premise for the thesis that bodies have no precise shape. To find support for the claim that all enduring states are vague, one therefore cannot appeal to the imprecision of shape in bodies. Other reasons will need to be tendered.

A natural idea might be that enduring states are vague because they are not precisely bounded in time. Perhaps it is unclear just when they begin and end. But it is far from obvious that Leibniz holds the relevant sort of unclarity to infect the duration of enduring states. We could put the question this way: for every possible enduring state, must there be moments in time that are 'borderline cases' of moments at which the enduring state obtains? It is hard to see why this should be true. Certainly for some enduring states there could be moments in time that are borderline cases of moments at which the state obtains. For example, suppose I have been poor for a while, but my poverty is gradually being removed by the successive addition of single pennies while writing this chapter; it may then be that this very moment is only a borderline case of a moment at which my enduring state of poverty still obtains. My enduring state of poverty may thus not be precisely bounded in time. But the vagueness of the enduring state in that example is clearly due not to its being an enduring state but rather to its being a straightforward case of vagueness tout court. It is not obvious that an enduring state can be vague simply because it is an enduring state. Enduring states that are vague seem as though they will always be vague for other reasons, and it cannot simply be presumed that those other reasons will be operative in every case of an enduring state.

Thus it seems that Leibniz has yet to produce plausible credentials for his claim that all enduring states are vague. Still, he does have solid grounds for saying that 'there is nothing precise about them'. Not all imprecision need be due to vagueness. The solid grounds he has for holding that there is nothing precise about enduring states are to be found in his view of the nature of time. So again we return to ideas arising in the Pacidius.
The fundamental difficulty for enduring states is that no interval of time can be assigned during which an enduring state can persist the same and uniform throughout. Leibniz is quite explicit that the motion of a body through any interval will suffer this infinite division; indeed, it is the very centerpiece of his theory of motion and his account of the structure of the continuum itself:

CHARINUS. Then what if we say that the motion of a moving thing is actually divided into an infinity of other motions, each different from the other, and that it does not persist the same and uniform for any stretch of time?

PACIDIIUS. Absolutely right, and you yourself see that this is the only thing left for us to say. But it is also consistent with reason, for there is no body which is not acted upon by those around it at every single moment. (A VI. iii. 564–5)

Just as a body's state of motion is never 'the same and uniform for any stretch of time', its shape undergoes constant change as well. Any stretch of time during which one shape might be assigned as 'the' shape of a given body is in fact divided into smaller subintervals of time during which the body actually takes on a number of distinct shapes. Likewise for each of the smaller subintervals, and so on ad infinitum. Thus the shape of the body does not persist the same and uniform through any stretch of time, but rather during every extended interval of time the body goes through an infinity of distinct transformations.

This again goes right to the heart of Leibniz's analysis of the composition of the continuum in which he tries to spell out a middle path between describing the continuum of time as 'purely continuous' and describing it as 'purely discrete'. The continuum of time is never undivided and uniform like a geometrical line, nor is it ever divided all the way into moments like the division of sand into grains. Instead, the divided continuum of time turns out to have a 'scaling' structure of intervals within intervals like 'a tunic scored with folds multiplied to infinity' (cf. A VI. iii. 555), never bottoming out into a mere collection of moments. This account of the continuum of time, space, matter, and motion describes a structure that can be recognized from a contemporary perspective to be fractal—or something very closely akin to fractal structure. Fractals are, broadly speaking, geometric objects that have structure on all scales of magnification, and, although the modern science of fractal
mathematics came together largely in the pioneering work of Benoît Mandelbrot in the latter half of the twentieth century, the ‘pre-history’ of fractal mathematics can be traced well back into the mathematics of the eighteenth century and indeed into Leibniz’s seventeenth-century writings. Leibniz’s own mathematical writings are shot through with insights that anticipate fractal mathematics both in large impressionistic gestures and in some fine technical details. What is most remarkable about the connection with fractal structure here is how perfectly it fills out Leibniz’s critique of the Cartesian theory of corporeal substance. If the actual bodies are fractally divided, then it is indeed impossible for any traditional geometrical shape to describe them. (Even the simplest cases of fractal structures, such as the Koch curve, can be described only as limits of infinite series of traditional shapes; and such constructions, though something Leibniz could perhaps glimpse, fall beyond the reach of the Cartesian geometry of extension and shape.)

Again, the details underlying this account are beyond the compass of the present chapter. But the basic line of thought that time is ‘fractally divided’ into intervals within intervals will illuminate the grounds for Leibniz’s key premise that ‘there is nothing precise’ about enduring states.

Because of the fractal structure of time, the very idea of an enduring state turns out to be an imprecise notion, not because it is a vague notion, but because of its inexactness as a representation of the nature of persistence through time. A body would have an enduring shape, for example, only if there could be some assignable stretch of time through which its shape persists the same and uniform. But in fact, through every such interval, any ‘precise and fixed’ shape can offer only an approximate guide to the actual shape of the constantly transforming body. Notice that this will hold no matter whether the purported ‘precise and fixed’ shape of a body is supposed to be fractal or ‘merely’ geometrical. Any interval of time through which a

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single shape is said to be the enduring shape of the body is actually divided up into several subintervals during which the body takes on a number of different shapes. Likewise for other supposed enduring states. They one and all involve something imaginary in the same way. Thus the very notion of an enduring state is only an imprecise notion relative to the actual course of changes in things through time, for the actual states of things can never be described with perfect accuracy by the notion of an enduring state. The notion of a state that endures through some interval can offer only an approximation, always less than perfectly correct, of what actually happens over the course of any stretch of time. And so, in this way, the notion of an enduring state is an inexact notion.

Finally Leibniz's main thesis from 'No Perfect Shape' can be defended and what we have been calling his 'secondary argument' from the nature of time and change for Proposition (I)—that there are no precise shapes in things, because of the subdivision of the continuum to infinity—can be brought fully to light. There can be 'no precise and fixed shape in things' and 'no moving thing of a definite shape', because bodily shape has to be understood as an enduring state. But, when held up against the reality of constant flux in the material plenum and the fractal division of the continuum of time itself, the notion of an enduring state proves to be only an inexact one. This means that there is nothing that can be accurately described as having an enduring state; and hence there is nothing that can be accurately described as having a particular shape.

On this reading of his argument, Leibniz's claim that there are 'no precise and fixed shapes in things' is vindicated, as is his claim that there is nothing precise about enduring states. The theories of time and change that are the legacy of Pacidius Philalethi make sense of his argument in 'There Is No Perfect Shape at all in Bodies', and give it solid grounds (or grounds that Leibniz could reasonably suppose to be solid, though the 'fractal theory' he holds is in fact not without difficulties of its own16). On this reading of Leibniz, however, his specific assertion that enduring states are 'vague', if that is meant to invoke the Sorites, will be contravened in favor of the claim that enduring states are inexact; and thus the present account

16 See my 'The Interval of Motion', and footnote 18 below.
of his argument against the existence of precise shapes in things may be a touch revisionary with respect to what Leibniz actually says. (At any rate, the claim that enduring states are ‘vague’ has been interpretively clarified here in a way that Leibniz himself does not clarify it.)

Still, his final example in ‘No Perfect Shape’ also seems to accord best with the claim of inexactness rather than vagueness. Just after concluding that there can be no moving thing of a definite shape, Leibniz writes:

For example, it is impossible for there to be found in nature a perfect sphere that would compose a moving body in such a way that this sphere could be moved through the least space. In a pile of stones, one could easily conceive an imaginary sphere that passes through all these stones, but one could never find any body whose surface would be precisely spherical. (A VI. iv. 1613–14)

What is the force of saying that one could never find any body whose surface is precisely spherical? Nothing in Leibniz’s metaphysics of matter could underwrite the claim that a body might have only an indeterminate and vague surface boundary. It is one of his most important and most frequently asserted metaphysical doctrines that actual bodies and the material plenum they form are everywhere fully determinate (cf. A VI. iii. 495; GP iv. 567; GP ii. 268, 278–9, 282; G vii. 562–3, etc.). Thus, when he says that one could never find any body with a precisely spherical surface, this is not because bodies are only vaguely bounded, but because the actual surface of any given actual body is determinately not spherical. To describe an actual body as having a spherical surface can give only an approximate account of its actual surface—approximate, but not exact. And likewise for all other traditional geometrical descriptions.

Let us at last sum up Leibniz’s secondary argument, from the nature of time and change, for the claim that there are no precise shapes in bodies, because of the division of the continuum to infinity. Bodies in the plenum are undergoing constant change, but the analysis of change demands that there be no middle moment or momentary state of change; thus change always occurs over a stretch of time, and in fact it consists in an aggregate of two moments containing two opposite states. Since the continuum of time cannot be composed
of isolated moments, there can be no moments in time apart from
the beginnings or ends of extended intervals of time. Hence no state
that is only momentary or exists only for a moment can be assigned
to a body, since that would require the existence of an isolated
moment that is not the beginning or end of an extended interval.
And so, in particular, no precise shape that is only momentary can
be assigned to a body; hence, a precise shape can be assigned to a
body only if an enduring state can be assigned to it. Because of the
fractal division of time, however, the very idea of an enduring state
turns out to be only inexact and imprecise, and therefore the very
idea of a shape that can be assigned to a body is only an inexact
and imprecise notion that does not apply to any actual body as it is
absolutely or in itself. Thus there are no precise shapes at all in actual
bodies.

So it is against the backdrop of his 1676 inquiry into the nature
of time, change, and shape that Leibniz finds the Cartesian theory
of body to rest on a ‘poor understanding of the composition of
the continuum’. The Cartesian theory had defined bodies in terms
of shape, but now, with a newly enlightened understanding of the
continuum, it appears that the very notion of shape cannot accurately
describe anything as it is absolutely or in itself, and thus the notion
of body that it defines can be only ‘imaginary’—that is, one of those
notions ‘that are not in things outside of us, but whose essence it
is to appear to us’ (A VI. iv. 70). The Cartesian theory of body
cannot offer an account of the true nature of actual bodies as they
are in themselves; and thus it collapses as a metaphysics of corporeal
substance.

5.

If Leibniz is right that there are no precise shapes in bodies, and that
the notion of shape that can be assigned to bodies is only imprecise
and imaginary, then the Cartesian theory of corporeal substance
stands refuted—as would any theory that rests so heavily on the
‘geometrical’ concepts of extension and shape. ¹７ But it seems that

¹⁷ My thanks to Dan Garber, whose discussion helped me to clarify this section
considerably.
Leibniz’s considerations about the notion of an enduring state will have far more profound consequences than just the defeat of the Cartesian conception of body. It is important here to consider the difference between the primary argument, which says that there are no precise shapes in things because assignable shapes are geometrical while bodies are fractally divided, and the secondary argument, which says that shape must be an enduring state but that time is fractally divided.

If, as the primary argument suggests, the problem with the notion of shape is that actual bodies are fractal in form while assignable shapes are only geometrical, then we could at least in principle have a theory of bodies that substitutes the fractal account of bodily shape for the old geometrical one, even if this means that the shapes of actual bodies are unassignable by us and that we can never even in principle grasp the true form of any particular body. The resulting account thus would not achieve the high hopes of the Cartesian project to found the metaphysics of body on concepts that are clearly and distinctly understood. Yet those would have been false hopes from the start. Leibniz stresses in his criticisms of Descartes that the latter fails to consider the difficulties of the labyrinth of the continuum, and so fails to appreciate the difficulties with the very

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18 Presumably Leibniz would seek to reconcile this deviation of the real shapes of things from the precise assignments that our geometry can offer by suggesting that the error of our assignments can always be made ‘smaller than any given error’, even if they can never be perfectly accurate in describing (fractal) reality. The ‘approach’ or ‘approximation’ of mathematics to reality is a staple feature of his accounts from his early writings right through the rest of his career. As he writes in De organo sive arte magna cogitandi (c. 1679): ‘Even if no straight lines or circles can be assigned in nature, it is nonetheless sufficient that shapes can be assigned that differ from straight lines and circles so little that the error is less than any given error—which is sufficient in order to demonstrate certainty as well as usage’ (A VI. iv. 159). See Beeley, ‘Mathematics and Nature’, 131. See also Arthur’s ‘Introduction’ to LOC. Leibniz’s view appears to be divided by two competing intuitions: the first, that actual curves will ‘converge towards continuity’, the second, that actual curves are ‘fractally divided’. Leibniz apparently wants to have it both ways, but those two intuitions will be difficult to reconcile.

19 Still, it is worth noting that, as early modern mathematics began to pursue ideas beyond the compass of Cartesian geometry, it was indeed exploring a dark and paradoxical subject; Descartes was not totally unjustified in drawing the official boundaries of geometry at algebraic curves—the only curves that he was confident admitted of ‘precise and exact measurement’—and withholding from the nascent infinitary methods the same claim to securing a distinct understanding of reality that he attributed to geometry. See Mancosu, Philosophy of Mathematics, 72 ff, and 141–5; and Grosholz, Cartesian Method, 42 ff.
notion of extension. If Leibniz is right about the nature of those difficulties and what is required to overcome them, then the Cartesian geometrical conception of body offers only an illusion of transparency to the mind, whereas the true understanding of the nature of body is far more difficult to reach, and indeed it cannot be achieved without negotiating a whole maze of paradoxes concealed behind the thought that mathematics will be a guide to a truly solid metaphysics.

If Leibniz’s primary argument against the existence of precise shapes in things poses a sharply focused challenge to the geometrical conception of body, and offers a visionary new account to replace it, his secondary argument from time and change is quite another matter. The charge that the notion of an enduring state is only imaginary, because time itself is fractally divided, raises a much more serious difficulty, and one that is hardly unique to the theory of Descartes. Perhaps the idea of fractal structure might coherently replace the traditional notion of geometrical shape and yield a new account of body; but what notion is going to replace the idea of an enduring state? It is not at all clear that there is a parallel replacement to be made here. No new conception of an enduring state seems to be forthcoming to take over where the old one failed. It is quite clear on Leibniz’s theory of the continuum that no state can be merely momentary; now it seems that no state can endure through time either. Yet what third alternative could there be?

It appears that the consequences of this secondary strand of Leibniz’s critique of the Cartesian theory of corporeal substance are very far-reaching indeed. Not only will no body that is defined in terms of magnitude, shape, and motion be truly substantial, but also nothing that is supposed to have either momentary or enduring states will ever be more than imaginary. Reality itself must somehow slip these bonds of time and change altogether. Perhaps our grasp of the nature of time and change is indistinct and rests, as Leibniz suggests, on a poor understanding of the composition of the continuum. But, even with his new theory of the continuum in hand, it is not clear that

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20 Descartes was not the only predecessor to suffer this criticism from Leibniz. In the *Passius*, just after announcing the necessity of solving the difficulties of the labyrinth of the composition of the continuum, Leibniz writes: "Neither Aristotle nor Galileo nor Descartes was able to avoid this knot, although one of them pretended it didn’t exist, one abandoned it as hopeless, and the other broke off his discussion" (A VI. iii. 547).
any coherent account of such a reality behind the appearances might be offered—nor that we could understand such an ‘enlightened’ theory, even if one were actually proposed. For, we might begin to suspect that, apart from time and change, reality can neither be nor be distinctly conceived.  

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