Chapter 3. Special Relativity and the lapse of time

1. Introduction: Gödel’s first argument for the unreality of time

If most people know one thing about Einstein, it is that he was the inventor of the theory of relativity. Many people have only a vague idea of what that theory is. In the popular press of his time it was thought to be the theory that “everything is relative”. But those who understand a little more know that the main novel feature of Einstein’s theory of Special Relativity (hereafter SR) is the claim that simultaneity is relative. What is simultaneous with me-now when I am at rest will no longer be simultaneous with me-now if I am moving with great speed. If I take myself to be at rest, then a particular event, such as a comet crashing into Jupiter, might have occurred at 3 p.m., according to the clock in my observatory (and this is something I could calculate on seeing that event through my telescope some minutes later, and calculating how long the light had taken to reach me from Jupiter’s known position). But if we take a frame of reference in which the Sun is stationary, then I am travelling on Earth at a sufficient speed that I traverse the whole orbit of the Earth around the Sun in a year: that is, at over 100,000 km/h, in a roughly straight line or inertial motion. Now, according to Einstein’s theory, from the frame of reference in which I and my laboratory are moving at over 100,000 km/h, there will be a different set of events that are simultaneous with my looking at the clock at 3 p.m., and this will no longer include the comet crashing into Jupiter, which will have occurred at an earlier or later time, depending on my direction of motion. In fact, by varying the speed of the reference frame in which we view my lab clock in one direction or another, and making it closer and closer to the maximum velocity c (the speed of light in a vacuum), the comet crash can be made to occur arbitrarily large times before or after 3 p.m. in that reference frame.

In short, there is, according to relativity theory, no unique set of events that are simultaneous with a given event, such as me at looking at my clock at 3 p.m. That will depend on the reference frame from which we consider my looking at it. So there is no unique ‘now’, or world-at-an-instant. For many authors, that “seals the deal” for temporal becoming. Time cannot consist in the successive existence of objectively defined worlds-at-an-instant if there are no unique worlds-at-an-instant.

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1 Here I am taking the Earth’s orbit as roughly circular and of a radius R = 93 million miles or 150 million km, the Earth travels \(2\pi R \approx 942.6\) m km in a year, for which the average speed would be about 107,600 km/h. (In comparison we can ignore the Earth’s relatively slow rotation on its axis, at roughly 1000 mph or 1670 km/h at the equator.)
One of the first to make this argument was Kurt Gödel, in the course of a paper he wrote in 1949—a paper more famous for the fact that here the prospect of time travel back into one’s own past was first raised as a possibility consistent with Einstein’s General Relativity. We’ll come to that in due course, when we consider the implications of curved spacetime in chapter 6. As we shall see there, Gödel took the possibility of such time travel to show that the assumption of an objective time lapse should be abandoned altogether.

Less remarked upon is the argument Gödel gives for the same idealistic conclusion earlier in that paper, based on the Special Theory of Relativity. There he argues that the assertion that two spatially separated events A and B are simultaneous “loses its objective meaning, insofar as another observer, with the same claim to correctness, can assert that A and B are not simultaneous (or that B happened before A)” (557). This, he claims, allows one to construct “an unequivocal proof for the view of those idealistic philosophers who, like Parmenides, Kant and the modern idealists, deny the objectivity of change and consider change as an illusion or an appearance due to our special mode of perception” (557). The proof he gives runs as follows:

Change becomes possible only through the lapse of time. The existence of an objective lapse of time, however, means (or at least is equivalent to the fact) that reality consists in an infinity of layers of “now” which come into existence successively. But, if simultaneity is something relative in the sense just explained, reality cannot be split up into such layers in an objectively determined way. Each observer has his own set of “nows”, and none of these various systems of layers can claim the prerogative of representing the objective lapse of time. (557-8)

Gödel’s idealistic conclusion is of course a radical one, and has not gained wide acceptance. According to the dominant view in the philosophy of science in recent decades, time intervals as measured in the various possible frames of reference are all perfectly objective, even if they are not invariant. Relativity of the duration of a process, it is argued, no more entails its subjectivity or illusory nature than relativity of the mass of a system to frame of reference entails the subjectivity or illusory nature of mass. The relativity of simultaneity entails that no one of these relative times can be privileged as the “actual time”, just as Gödel had argued. Nevertheless, each measure of duration is consistently related to any of the others by the Lorentz transformation formulas. According to the dominant view—as subscribed to, for instance, by Jack Smart, Adolf Grünbaum,
Paul Davies, and many others—what is refuted by such arguments from the relativity of simultaneity is not the objectivity of time lapse, but the notion of coming into existence. It is true, as Gödel observed, that different choices of inertial reference frame will result in wholly different classes of events being simultaneous with a given event, and that one must therefore relinquish the classical notion of a world-wide “now”. What this precludes, however, is not the objectivity of time lapse, but—as Hilary Putnam (1967) and C. W. Rietdijk (1966) each argued on grounds similar to Gödel’s in the late 60’s—any notion of objective becoming (becoming real, in Putnam’s case, becoming determined in Rietdijk’s). (A similar argument was given by Nicholas Maxwell (1985).) Thus although time lapse is perfectly objective, it is frame-dependent.

Interestingly, Gödel, anticipated this objection that the relativity of time lapse “does not exclude that it is something objective.” To this he countered that the lapse of time connotes “a change in the existing”, and “the concept of existence cannot be relativized without destroying its meaning completely” (558, n. 5). The dominant view, by contrast, would urge that the relativity of existence is avoided precisely by denying that time lapse constitutes a “change in the existing”: the existence of events is their existence in a four-dimensional spacetime, and this does not change. Against this, as has been argued elsewhere, the sense in which spacetime “exists” is not a temporal sense, and so will not support the contention of Putnam et al. that events simultaneous with another event are “already real” for it. To suppose that this is so, I argue, leads inexorably to a conclusion that denies the reality of temporal succession.

What I wish to draw attention to here, however, is a premise that the dominant view shares with Gödel’s: both assume that events are real or determined when they are present to an observer, with presentness construed in terms of simultaneity in the observer’s frame of reference; i.e. they construe the reality of an event in terms of the time co-ordinate function. Thus Putnam (1967) and Rietdijk (1966) each assume that becoming real or determined must occur relative to “an observer’s inertial system”, with time-lapse measured by the time co-ordinate function, as a premise in their reductio arguments against the reality of becoming real or determinate. The crucial premise here is the Gödelian one that for each individual observer, “the existence of an objective

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2 See Smart (1968, 255ff.) and (1980); Grünbaum (1976); Davies (1989, 3): “Thus relativity physics has shifted the moving present out from the superstructure of the universe, into the minds of human beings, where it belongs… present day physics makes no provision whatever for a flowing time…”.

lapse of time … is equivalent to the fact that reality consists in an infinity of layers of ‘now’ which come into existence successively.” That is, the time lapse between, for instance, two events in anyone’s life history is given by the difference in the values of the time co-ordinate function in some particular inertial reference frame.

In this chapter I shall argue that such characterizations of time lapse are fallacious precisely because they keep fused together two distinct functions of time that are (degenerately) fused in the classical conception of time: (a) synchronization, or determining which events are simultaneous with a given event in a given reference frame, and (b) invariant time lapse. In SR the becoming of events in succession, the rate of a process or the rate at which a thing ages, is tracked by proper time \( \tau \); the synchronicity of distant events is tracked by the time-coordinate function \( t \). This separation of these two different aspects of time into two different time concepts \( \tau \) and \( t \) is characteristic not only of SR, but of all relativistic physics, where every timelike curve represents a possible process, whose rate of evolution is parametrized by proper time. But in order to understand these concepts—inertial reference frame, the Lorentz transformation formulas, proper time, timelike curves, and so forth, we need to take a step backwards, and consider the creation of the Special Relativity by Albert Einstein and Hermann Minkowski.

2. Reference frames, Einstein’s Machian approach, and Minkowski’s spacetime

{* In this section I will introduce the idea of reference frames, and show how relationalism changed its meaning from Huygens’s idea that velocity is relative to body to the idea that it is relative to a co-ordinate system;

how the influence of Mach’s positivism motivated Einstein to give a construction of “at the same time as” in terms of what measurements would support it; Lorentz’s “local time” Einstein’s time coordinate.

something on A. A. Robb’s opposition to Einstein, and how he gave a construction of the special theory of relativity in terms of the causal connection and instants

how Minkowski reconstructed SR in terms of spacetime and thought it would have been better to call it the theory of invariants (Invariantstheorie); his introduction of proper time; and how others showed that the light postulate is redundant. *}
3. Rietdijk, Putnam, Maxwell

Now let us look at some of the arguments from the relativity of simultaneity to the reality of all events in the manifold. We have seen that SR rules out the idea of a unique, absolute present: if the set of events that is simultaneous with a given event \( e \) depends upon the inertial reference frame chosen, and in fact is a completely different set of events (save for the given event \( e \)) for each choice of reference frame in inertial motion relative to the original, then there clearly is no such thing as the set of events happening at the same time as \( e \). In the vivid example of Paul Davies, if I stand up and walk across my room, the events happening “now” on some planet in the Andromeda Galaxy are different by a whole year than those that would be happening “now” if I had stayed seated. (Davies, 1995, 70). This much is clear and uncontroversial. But from it Davies concludes: “unless you are a solipsist, there is only one rational conclusion to draw from the relativity of simultaneity: events in the past and future have to be every bit as real as events in the present… To accommodate everybody’s nows, … events and moments have to exist all at once across a span of time.” (1995, 71)

But this is by no means a rational conclusion to draw. Events “exist all at once” in a spacetime manifold only in the sense that we represent them all at once as belonging to the same manifold. But we represent them precisely as occurring at different times, or different spacetime locations, and if we did not, we would have denied temporal succession. The rational conclusion to draw, I submit, is that (according to Special Relativity) distant events that are simultaneous with some given event—for example, the event of my considering them—cannot be supposed to be ‘real’ or ‘existent’ for that event, e.g. existent for me at the spacetime location from which I am considering them.

More elaborate arguments along the same lines as Davies’ had previously been given (in papers written independently at nearly the same time) by Putnam (1967) and Rietdijk (1966). Although the details of their arguments differ, both depend on a scenario that can be described as follows. We are asked to imagine two spatially distant inertial observers, \( O_1 \) and \( O_2 \), with one moving at an appreciable fraction of the speed of light with respect to the other. At a certain time according to the observer \( O_1 \)’s own inertial system, an event \( b \) that is happening to \( O_2 \) is “present” or “now” for \( O_1 \), and we may imagine \( O_1 \)’s being aware of this as the event \( a \); but to \( O_2 \), the event happening to \( O_1 \) that is simultaneous with \( b \) in her inertial system is not the event \( a \), but another event \( p \). Yet it is easy to set the relative velocity in such a way that \( p \) is in the future for \( O_1 \) at the time that he is
experiencing $a$. It follows that, if all those events are real which are present for a given observer in that observer’s inertial system, then $b$ is real for $O_1$ when he is experiencing $a$, and $p$ is real for $O_2$ when she is experiencing $b$. Thus if $xRy$ denotes “$x$ is real for $y$” we have $bRa$ and $pRb$, so that, if $R$ is transitive, then $pRa$ (“$p$ is real for $a$”) even though $p$ is in the future for $O_1$ when he is experiencing $a$. We are forced to conclude, reasons Putnam, “that future things (events) are already real” (Putnam 1967, 242), or as Rietdijk puts it, “that, being ‘past’ or ‘present’ for only one inertial system, an event can be shown to be determined in all other systems” (1966, 342), so that “there is determinism” and “there is no free will” (343).

Putnam, it should be said, acknowledges that simultaneity, although transitive within any given frame of reference, is not transitive between frames: “the relation ‘$x$ is simultaneous with $y$ in the co-ordinate system of $x’$ … is not transitive” (242-43). So he does not claim that all events exist “at once” in the sense of being mutually simultaneous. Nevertheless, he argues, the assumption that “all things that exist now according to my co-ordinate system are real”, in combination with the principle that “there are no privileged observers”, requires the relation $R$ to be transitive (243). But if $R$ is to be interpreted to mean that future events “already exist”, as Putnam asserts, then this is to imply that they have, as of the earlier time, already occurred. A similar criticism applies to Rietdijk’s conclusion: an event $p$ can only be said to be “already ‘past’ for someone in our ‘now’” (341) at location $a$ in the sense that it has already occurred at $a$. But such a claim amounts to a denial of temporal succession.

In each case we are presented with an argument that begins with a premise that all events existing simultaneously with a given event exist or are real, and concludes that consequently all events in the manifold are real. But the conclusion only has the appearance of sustainability because of the equivocation analysed above. If a point-event exists in the sense of occurring at the spacetime location at which it occurs, it cannot also have occurred earlier. But if the event only exists in the sense of existing in the manifold, then the conclusion that it already exists earlier—that such a future event is “every bit as real as events in the present” (Davies), or “already real” (Putnam)—cannot be sustained. Thus, far from undermining the notion of becoming, their argument should be taken rather to undermine their starting premise, that events simultaneous with another event are already real or already exist for it in a temporal sense. For to suppose that this is so, on the above analysis of their argument, inexorably leads to a conclusion that denies temporal succession.
What I wish to draw attention to here, however, is a premise that the dominant view shares with Gödel’s: both assume that events are real or determined when they are present to an observer, with presentness construed in terms of simultaneity in the observer’s frame of reference; i.e. they construe the reality of an event in terms of the time co-ordinate function. Thus Putnam (1967) and Rietdijk (1966) each assume that becoming real or determined must occur relative to “an observer’s inertial system”, with time-lapse measured by the time co-ordinate function, as a premise in their reductio arguments against the reality of becoming real or determinate. The crucial premise here is the Gödelian one that for each individual observer, “the existence of an objective lapse of time … is equivalent to the fact that reality consists in an infinity of layers of ‘now’ which come into existence successively;” That is, the time lapse between, for instance, two events in anyone’s life history is given by the difference in the values of the time co-ordinate function in some particular inertial reference frame.

But this construal of time lapse in SR is false, as can be shown by an analysis of the much discussed Twin Paradox. Here we imagine one twin staying at home while the other speeds off at a relative velocity which is an appreciable fraction of c, the speed of light, turns round, and returns at a similar velocity. When they are reunited, less time has elapsed for the travelling twin, who is consequently found to have aged less. But the discrepancy between the times elapsed for the two twins cannot be a discrepancy between times as measured by co-ordinate time —the time or “layer of ‘now’” associated with some given inertial system— since in that inertial frame of reference the twins are apart for exactly the same time, as measured by the time co-ordinate of that frame.

Indeed, in any such inertial frame, there is only one difference between the co-ordinates of these two points, and not one for each twin. In fact, the time taken for the twins to make each of their trips through spacetime from the point at which the travelling twin departed to the later point of their reunion must instead be determined by integrating the proper time along each twin’s particular world line. Thus the root of the trouble with the “layer of now” conception of time lapse is a failure to take into account the degeneracy of time. Time lapse is measured by the proper time. The difference in the proper times for their journeys is not the same as the difference in the time co-ordinates of the two points in some inertial reference frame. If time lapse were measured by such a time co-ordinate function, then both twins would be the same age. They are not. Ergo, time lapse (in the sense of how long a given process takes, how quickly it becomes) is not measured by
the time co-ordinate function. So Gödel’s “unequivocal proof” of the ideality of time falls flat on its face.⁴

It is puzzling that this simple consideration is not widely recognized; this suggests that there are other assumptions at work that mask its application. I believe they have to do with a misconception of proper time as the time co-ordinate of the observer’s rest frame, and related misconceptions about an observer “inhabiting an inertial frame” and “experiencing” the events which are simultaneous with his or her state of consciousness. Rietdijk, for example talks of two spatially separated observers “experiencing the same present ... in virtually the same inertial system” (1966, 342), Grünbaum writes that for an organism M to experience an event at a time t is to be “conceptually aware of experiencing at that time either the event itself or another event simultaneous with it in M’s reference frame” (1976, 479), and Putnam of “everything that is simultaneous to you-now in your co-ordinate system” being real, and Clifton and Hogarth of two observers’ “inhabit[ing] the same inertial frame” (1995, 379). Although misconceptions about proper time are seldom stated explicitly, they also appear to be quite prevalent. Indeed, they afflict the understanding of SR itself, as witnessed by some of the attempted resolutions of the Twin Paradox.

These considerations motivate another look at the Twin Paradox, to get clear on what is in an observer’s (visual) experience in a relativistic context, and what is inferred; and to see more clearly how the distinction between proper time and co-ordinate time cleanly resolves the paradox without reference to the events one “experiences” as present undergoing a dramatic change, (Section 4) or implying that the discrepancy in the twin’s ages is a General Relativistic effect (Section 5). This may seem otiose, given the number of times the paradox has been resolved, and given that no one who knows relativity thinks it a problem. But anyone who believes that the resolution of the paradox requires General Relativity, or a recognition that the events experienced as present by the moving twin undergo a discontinuous shift at the point of return, or that there is any asymmetry in what speeding up or slowing down of clocks is seen by or inferred about either twin of the other, has not properly appreciated, so I would claim, the profundity of the changes in our understanding of time wrought by Special Relativity.

⁴ It may be countered, as by my anonymous referee, that Gödel’s argument depends only on the lapses of time being different for any two arbitrary curves connecting two timelike related events, and that Gödel does not assume that time lapse is measured by a time-coordinate function. But Gödel explicitly construes time lapse in terms of co-ordinate time in his argument from Special Relativity, where his argument against the “relativization of existence” crucially depends on this. This is supported by the interpretation of Palle Yourgrau (1991), who construes Gödel’s argument as depending on a conception of time lapse as relative to reference frame.
4. The Twin Paradox Revisited

To make our well-worn paradox vivid, let us take Terence the true Tellurian, who tethers himself to terra firma; and Astrid the astronaut, who abridges her age by abandoning Earth with alacrity for Alpha Centauri. (Since the tale of the two twins has been told so many times, I hope I may be allowed a little alliteration in the account.) We’ll assume, to keep the figures round, that Alpha Centauri is 6 light years away, and that Astrid approaches it at six tenths of the speed of light (.6c), turns round in an instant, and returns towards Earth at the same speed (all of this in Terence’s rest frame, i.e. from the point of view of an inertial frame of reference in which Terence is stationary). An easy calculation shows that, according to Terence, his twin is away for exactly twenty years (ignoring for now any periods of acceleration or deceleration). Things are otherwise for Astrid. At such great speed, distances are foreshortened by a factor \( \sqrt{1 - 0.36} = \sqrt{0.64} = 0.8 \): it appears to her that she makes a journey outwards of only 4.8 light years, and does it in 8 years. (Seen from Terence’s perspective, time runs more slowly on her watch.) An equal space contraction and time dilation occurs on her way home, so that she takes only 16 years for her journey (still discounting the time of her deceleration). Thus when they are reunited and compare their watches, they find that Terence has aged 4 years more than his sister! This seems obviously paradoxical. If all inertial motion is relative, how can there be an absolute difference in their lifetimes resulting from it? The paradox is heightened by this observation: while the twins are in relative inertial motion, each’s duration will be running slower from the perspective of the other’s rest frame. In each leg of the journey, Astrid would infer processes to be happening more slowly on Earth as it receded from her or approached her at 0.6c: her eight years would correspond to an inferred duration of processes on earth of only 6.4 years! As Herbert Dingle reasoned, if each twin’s life-process is slowed down relative to the other, each will age less than the other, an obvious impossibility! This is the paradox.

A standard resolution given of this paradox explains that the reason for the discrepancy is that as it is only Astrid who undergoes an acceleration as she turns around, it is therefore she who performs an absolute motion, not Terence. On this analysis, so long as the twins are in inertial motion relative to one another, each twin must indeed infer that the other’s clock is running slow. The reason for the discrepancy is that, as Astrid turns about, the act of her deceleration skews her

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5 Herbert Dingle (1890-1978) was an English astronomer who wrote a standard textbook on relativity theory before becoming a vociferous opponent in his old age. See (Dingle 1972).
temporal orientation violently, and under the conditions stated, instantaneously. As she journeys home she infers Terence’s clock to advance only 6.4 years, yet it will read 20 when she returns to Earth. So, at the time she sets out from Alpha Centauri for the journey home, it must read 13.6. Yet the instant before, the instant she arrives at Alpha Centauri, she would have inferred it to have read 6.4 years! So, instantaneously, it would have had to have jumped 7.2 (= 13.6 – 6.4) years.

So it’s not that Astrid’s instantaneous (and wholly unphysical) acceleration introduces a time dilation; it’s that it discontinuously skews her temporal orientation. Now if we were to follow Putnam’s informal way of speaking, we would say that Astrid “experiences” 7.2 years going by in an instant: events that were “present according to her co-ordinate system” are discontinuously displaced 7.2 years into the past of “her-now” according to that same system. In actual fact, however, no such wrenching change of her experience of the present occurs. These are facets of a sloppy use of the ideas of “observer’s reference frame” and the observer’s “present”, and a failure to distinguish between the time an observer might infer an event to occur from when the observer would see it occurring. In fact, it will be worth going over the whole thing in some detail to see how this could be the case. Taking my cue from the lucid account of the twin paradox given by Paul Davies in his recent book, I shall re-examine the thought experiment by conducting it in three stages.

First, just to get our bearings, let us assume a Cartesian cosmos, in which light is a pression that is transmitted instantaneously, and durations are completely independent of the state of motion of the enduring thing. We equip each twin with a very powerful telescope and a very large clock, and then suppose Astrid to leave for Alpha Centauri at 0.6c. This is very straightforward. The Tellurian twin sees his astronaut sister arriving at Alpha Centauri 10 years after she left, and sees his sister’s clock register those 10 years. When Astrid arrives at Alpha Centauri she sees Terence’s clock register that ten years have passed, and sees him aging a year per year as she returns home.

Now let’s assume a Dopplerian universe. In this universe, it is known that Descartes was wrong not to have listened to his mentor Beeckman: light travels at a finite speed c (in Terence’s Tellurian frame of reference). Otherwise the universe is classical, as before. Now things are interestingly

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6 Cf. Lawrence Sklar (1974, 272): “One must always be careful in special relativity to distinguish what an observer actually sees, literally, from what he computes to be the case”.

7 Davies (1995, 59-65) considers the twin paradox by “equip[ping] our twins with a powerful telescope so that they can watch each other’s clocks throughout the journey,” and then discusses what clock readings they would see, distinguishing the Doppler effect from the time-dilation effect.
different for the interstellar twins: because light takes 6 years to travel from Alpha Centauri to Earth, when Terence actually sees the event of Astrid’s arrival there, 16 years will have passed since they parted! He sees Astrid’s clock register only 10 years while his has registered 16. (Astrid’s clock is apparently running slow by a factor 5/8 compared to his). He is then even more surprised to see his sister take only 4 years to return, and watches his sister’s clock running at 2.5 times the speed of his own; thus, as Terence views Astrid’s return trip and all the processes happening in it, he sees them appearing to occur four times as fast (2.5 divided by 5/8) as during the trip outwards! Astrid has an analogous experience. When she arrives at Alpha Centauri, she observes Terence’s clock to be reading only 4 years. For the image of the clock registering 4 years travels the 6 light-years to Alpha Centauri to arrive there 10 years later (all from Terence’s frame of reference). So Astrid sees Terence’s clock has been running 2.5 times as slowly as hers (i.e. at 2/5 speed)! But on the way home she sees it to be running 1.6 times as fast: in the ten years it takes Astrid to return, Terence appears to her to age 16 years! Thus she, too, is puzzled to see her twin’s clock going 4 times as fast (1.6 divided by 2/5) as it was on the outgoing leg of the journey. But the twins’ puzzlement is relieved when they learn about the Doppler effect: Events and processes occurring in a frame of reference in motion towards an observer appear to be speeded up (“blue shift”); occurring in a frame of reference in motion away from an observer they appear to be slowed down (“red shift”).

Still, this does not explain the discrepancy in their experiences. Granted there is a certain symmetry: each twin sees the other’s clock running 4 times as fast on the return trip as it was on the way to Alpha Centauri. But if all inertial motion is relative, they should have experienced the same red shift while moving apart, and the same blue shift when moving back towards one another. That they didn’t is explained by the fact that we have taken the speed of light to be $c$ in Terence’s frame only. If the speed of light is also $c$ in Astrid’s frame of reference, then the situations are entirely symmetrical, and Astrid should see Terence’s clock run slow on the outward trip by a factor of 1.6, and fast on the return journey by a factor of 2.5. Thus when she reaches Alpha Centauri, she should see Terence’s clock read 10 divided by 1.6 = 6.25 years. But then Terence would age $20 - 6.25 = 13.75$ years while she returns. This is a long way short of his aging calculated by the Doppler factor 2.5, 10 times 2.5 = 25 years, an obvious impossibility! In short, the assumption that the speed of light is the same in all frames of reference is at variance with the assumption of classical physics that all inertial motion is relative.
This, then, is the kind of discrepancy that physicists faced at the beginning of the twentieth century. All the experimental evidence seemed to suggest that the speed of light is the same in all inertial reference frames. But this is incompatible with the requirement that everything will appear the same from each inertial reference frame, unless something else gives. In terms of the twin example, the only way for Astrid to see a Doppler effect equal to Terence’s is if the length of the journey in a frame of reference in relative motion were to suffer a contraction along the direction of motion by a factor of \( \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - 0.36} = 0.8 \). (That the dimensions of a body are distorted in this way by their motion through the aether was independently suggested by George FitzGerald and Hendrik Lorentz,\(^8\) taken up by Joseph Larmor, and generalized by Henri Poincaré.) But this also implies a similar effect on the rates at which processes occur: they must slow up by the same factor in a frame of reference in relative motion. In terms of the present example, if the twin in motion at 0.6c covered a distance of only 4.8 light years (6 times 0.8) in her own frame of reference, then this would take her only 4.8 ÷ 0.6 = 8 years (= 10y times 0.8) in that frame. Time for the moving twin would, from the point of view of the stationary one, run more slowly by a factor of 0.8. This is the so-called time dilation effect, partially understood by Larmor and Lorentz, but first explicitly articulated by Einstein,\(^9\) and now known to be a really occurring effect. Lorentz and company, of course, assumed that all these dilations and contractions could be referred to the frame in which the aether is at rest, and would effectively prevent one from detecting which frame this is. This is where Einstein stepped in: he dispensed with the unobservable-in-principle aether, so that each inertial frame would be on the same ontological footing.

So let us assume, finally, an Einsteinian universe for our twins. As before, when Terence actually sees the event of his sister’s arrival at Alpha Centauri, his own clock registers 16 years. But, because of time dilation, Astrid’s clock registers only 8 years, and thus appears to Terence to be running slow by a factor of \( \frac{1}{2} \), i.e. 1/2 as fast as his. On the return leg, Terence sees his sister’s clock advance 8 years whilst his only advances 4; so the clock (and the aging processes of his sister and everything moving with her) appear to be running fast by a factor of 2. Despite these appearances, of course, Terence (who is now a whiz at physics) can infer that in each case the effect is 0.8 times

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\(^8\) Harvey Brown, in his (2005, 3, 45-55), explains that the conjecture of FitzGerald and Lorentz was that the dimensions of the body were altered in a certain ratio, not that there was a physical contraction along one of them. See also Mauro Donato, “Relativity theory between structural and dynamical explanations,” forthcoming in *International Studies in Philosophy of Science*, preprint p. 7.

\(^9\) For an authoritative discussion of the extent to which Larmor, Lorentz, and Poincaré did and did not anticipate Einstein’s discovery of time dilation, see Brown (2005), esp. ch. 4.
what would be expected from the Doppler effect alone: a lag by a factor of 5/8, multiplied by 0.8, gives 1/2; a speeding up by a factor of 2.5 times 0.8, yields 2. Thus he infers that Astrid’s clock is running slow because of time dilation.

Astrid, on the other hand, on looking back to Earth as she is arriving at Alpha Centauri 8 years later, sees the Tellurian clock register 4 years, as before. By Astrid’s reckoning, the image has travelled eight years to get to Alpha Centauri, so Terence’s clock appears to Astrid to be running slow by a factor of 2. On the return leg, Astrid sees her brother’s clock advance its remaining 16 years whilst hers only advances 8; so the clock (and the aging processes of her brother and everything moving with him) appear to be running fast by a factor of 2. Again, she can calculate that since the effects should have been 5/8 and 2.5 if they were due to the Doppler effect alone, the difference is due to the fact that Terence’s time is slowed relative to hers by the time dilation factor 0.8. (During her 8 years on the outward leg, she sees Terence’s clock move 4 years when it should have moved 5 by the Doppler effect alone, since 8 times the Doppler effect of 5/8 = 5; on the way back she sees Terence’s clock move 16 years instead of the 20 that would be 8 times the Doppler effect of 2.5). Thus the situation is entirely symmetrical: while they are in relative motion, each twin suffers an inferred time dilation, a slowing-down of the aging process, from the point of view of the other. And in terms of what appears, they both see their twin sibling’s clock running slow by a factor of 2 while they are moving apart, and running fast by a factor of 2 when they are approaching one another.

This shows us that the scenario depicted is entirely consistent. But how does it resolve the paradox? If everything is symmetrical, then why don’t the twins age by the same amount? The usual (and correct) explanation is as follows: although while the twins are in constant relative motion the situations are indeed perfectly symmetrical, this is not so for their journeys or paths through spacetime as a whole. For in this thought experiment, the terrestrial twin Terence undergoes no acceleration, whilst his adventurous astronaut sister must decelerate through –0.6c on arrival at Alpha Centauri, and then accelerate through another –0.6c to the same speed in the opposite direction. On the other hand, in the idealized conditions of the thought experiment, any time dilations due to the accelerations are ignored. But although this explanation is perfectly correct, it leaves a lingering sense of puzzlement. If the difference in the ages of the twins is not due to any time dilation caused by acceleration, and yet while they are in inertial motion relative to one another each sees the other’s time dilated by the same factor, how does the asymmetry in the paths
taken result in a difference of time elapsed for each twin? How is a difference in paths even relevant to the situation? To many people, this has suggested that the difference in the twins’ ages is due to a time dilation caused by acceleration.

5. Modified Twin Paradox

Thus it is often said that the reason for the discrepancy in the twins’ ages is that whereas Terence is in inertial motion throughout, Astrid is the one who really moves because of her acceleration, although this acceleration lies outside the scope of the theory. This seems to imply that it is Astrid’s (here instantaneous) non-inertial motion that is responsible for the dilation. Some even go so far as to claim or imply that Special Relativity applies only to systems in inertial motion, and that a proper resolution of the paradox must therefore involve General Relativity. But this is incorrect. On the one hand, Special Relativity is perfectly applicable to accelerated motions, and on the other, although the fact of Astrid’s acceleration is a necessary condition for their taking different paths through spacetime, the time dilation is due to the different paths, not an effect of the acceleration itself.

To see this, let us take a more realistic scenario, where Astrid takes a couple of years (as measured from an inertial Earth) to decelerate to a speed of zero as she reaches Alpha Centauri, and accelerate back up to the same speed in the opposite direction, undergoing a time dilation due to the acceleration as she does so. But let us give Terence an equivalent acceleration during the

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10 See for instance J. J. C. Smart, (1968, 231): “The clock paradox comes from the following fallacious bit of reasoning. In our calculations we have taken Jack to be at rest and Jim to be moving with a velocity of either +v or −v relative to him. Equally, it is said, we could take Jim to be at rest… The fallacy in the reasoning is that the first calculation (showing Jim to be younger than Jack) was correct, because Jack has been in the same inertial system throughout. However Jim had to be accelerated at Alpha Centauri…”

11 To cite two contemporary examples from the world wide web: “However, this resulted in a limitation inherent in Special Relativity that it could only apply when reference frames were inertial in nature…” (http://en.wikipedia.org/wiki/Inertia); “This dilemma highlights a limitation of the Special Theory of Relativity that we have already alluded to. It only applies to observers in uniform motion, and not to accelerated frames.” (http://theory.uwinnipeg.ca/mod_tech/node141.html).

12 Cf. this analysis on the Encyclopedia Britannica internet site: “The answer is that the paradox is only apparent, for the situation is not appropriately treated by special relativity. To return to Earth, the spacecraft must change direction, which violates the condition of steady straight-line motion central to special relativity. A full treatment requires general relativity, which shows that there would be an asymmetrical change in time between the two sisters. Thus, the “paradox” does not cast doubt on how special relativity describes time, which has been confirmed by numerous experiments.” http://qa.britannica.com/eb/article-252886.

13 One suspects that Einstein himself unwittingly contributed to this misunderstanding by using arguments in his (1918) from General Relativity to defend the consistency of the Special Theory. But in fact what Einstein is defending is not the self-consistency of SR attacked by Dingler, but the consistency of the account of time dilation due to accelerated motion in SR with the General Relativistic equivalence of acceleration with gravity.
same period. In this way we can construct a journey for each of the twins with equivalent non-inertial path. Thus (to resume the alliterative account) suppose the trusty Terence tires of his terrestrial tenure, and takes residence in Telstar, a nearby space-station, a doughnut shaped ship that simulates gravity by rotating. To relieve tedium, Terence sets it spinning very fast about an inertial point, so that for precisely the period of 2 years (in the terrestrial frame) in which Astrid is decelerating at \(-0.6c\) a year, and undergoing a corresponding time dilation due to this acceleration (leaving her \(D\) years younger, where \(0<D<2\), Terence undergoes an exactly corresponding time dilation due to his having undergone an equivalent rotational acceleration. Now when Astrid returns, she will be between 16 years and 18 years older, and Terence will be between 20 and 22 years older, and they will differ in age by exactly 4 years. In the reference frame of the inertial point near earth, exactly 22 years will have passed.\(^{14}\) But in their paths through spacetime, the twins will have been in inertial motion for the same 20 years with respect to that point and its inertial frame, and will have undergone time dilation due to their accelerations for the same two years in that frame. Yet one is still 4 years older than the other. It follows that it can’t be said that the difference in their ages is due to the time dilation resulting from their having been accelerated. Nor can it be said that Special Relativity applies only to systems in inertial motion — if that were so, there could be no explanation of the Twin Paradox in the theory. But we have just so explained it! Thus the difference between the ages of the twins is not due to one’s being in inertial motion, the other not. Both their ages are true measures of time, in the original thought experiment, as well as in this modified version.

The correct conclusion is that it is not any difference between inertial frame-times that accounts for the difference in the twins’ ages, but the difference in their paths through spacetime. It is the time elapsing along a particular path in spacetime that measures how fast the processes traversing that path are going, how fast the people or things undergoing them are aging, how fast they are becoming. In the non-Euclidean metric of Minkowski spacetime, it is the longest, not the shortest, time interval between two spacetime points that is given by the straight line in spacetime connecting them. The longer the spacetime path between them, the shorter the time elapsed along that path. In the original thought experiment, Astrid travels along two sides of a triangle, and Terence by the remaining side; in the modified version, Astrid’s straight lines are joined by a curve,

\(^{14}\) Of course, the physical situation could be made even more realistic, if desired, by having Astrid accelerate away from Earth to his speed of 0.6c, and then decelerate to zero on return. But again, this could be compensated for by having Terence spin his Telstar for the same period.
while Terence’s straight line is interrupted by a spiral of the same length. In each case Astrid’s path is longer, and the time elapsed shorter. It follows that it can’t be said, as one often reads, that the duration of processes in relativity theory is relative to an inertial frame. In the sense of time lapse that is relevant to the twin paradox—how much time elapses for each twin—it is simply false that time lapse is frame-dependent, i.e. depends on the inertial frame adopted. Indeed, the duration of each twin’s journey through spacetime is an invariant measure: it is the same in all inertial reference frames.

6. Proper Time and Proper Length

As I stated at the start of this chapter, there are in fact two different measures of time in relativity theory: they have different formal measures, and different ontological baggage. This parallels the case for mass. In each case, what in classical physics had been thought to be a univocal or absolute property of the system turned out to be degenerate. For in the transition to Einstein-Minkowski physics mass bifurcates into the relativistically invariant proper mass $m_0$, and the relative mass $m$, or mass in an inertial frame in which it is moving at a speed $v = \beta c$, whose quantity is a factor $\gamma = \left(1 - \beta^2\right)^{-1/2}$ times the proper mass.

But, as I have suggested above, I believe much of the confusion about relativity comes from interpreting proper time as if it is simply the relative time of an observer in her own rest frame. This misinterpretation is encouraged by the analogy with mass, but even more so, I will now suggest, by reading the case of time or duration as an exact analogue of that of space or length. For the degeneracy of time in relativity theory is paralleled by a similar bifurcation in the concept of length. A body that is moving at a speed $v = \beta c$ with respect to a given inertial reference frame will, as already discussed, undergo a length contraction in the direction of its motion, so that its length $L = L_0 / \gamma$, where $L_0$ is its proper length. The latter is defined as its length in the rest frame: if $v = 0$, $L = L_0$. Analogously, it may be thought, any periodic processes associated with the body will suffer a time dilation, so that $t = \gamma t_0$, with the result that in the rest frame where $v = 0$, $t = t_0$. Proper time, then, it may be asserted, is just $t_0$, the time coordinate as measured in the rest frame.

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15 I am indebted to Storrs McCall (private communication) for suggesting to me the relevance here of the analogy with proper length. I am also indebted to Kent Peacock for helping me eradicate some infelicities in my discussion of this in an earlier draft.
But this it is not! Proper time was introduced by Hermann Minkowski in his famous 1908 paper (Lorentz et al. 1923, 73-91) as follows. If at any point P \((x, y, z, t)\) in spacetime we imagine a worldline running through that point, the magnitude corresponding to the timelike vector \(dx, dy, dx, dt\) laid off along the line is

\[d\tau = \sqrt{(c^2 dt^2 - dx^2 - dy^2 - dz^2)}/c\]

Proper time is now defined as the integral of this quantity along the world line in question. Introducing the concept, Minkowski wrote: “The integral \(\tau = \int dt\) of this quantity, taken along the worldline from any fixed starting point \(P_0\) to the variable endpoint \(P\), we call the proper time of the substantial point at \(P\).” (85) As he proceeded to explain, \(x, y, z\) and \(t\)—the components of the vector \(OP\), where \(O\) is the origin—are considered as functions of the proper time \(\tau\), and the first derivative of the components of this vector with respect to the proper time, \(dx/dt, dy/dt, dz/dt\) and \(dt/dt\), are those of the velocity vector at \(P\).

It is a consequence of this definition that the element of proper time \(dt\) is not a complete differential. Arnold Sommerfeld, in his notes appended to Minkowski’s 1908 paper when it was reprinted in a book (Lorentz et al. 1923, 92-96), remarked that Minkowski had mentioned this to him. He comments:

\[\text{T}he \ element \ of \ proper \ time \ \int dt \ is \ not \ a \ complete \ differential. \ Thus \ if \ we \ connect \ two \ world-points \ \(O\) \ and \ \(P\) \ by \ two \ different \ world-lines \ 1 \ and \ 2, \ then\]

\[\int_1 \ d\tau \neq \int_2 \ d\tau\]

If \(1\) runs parallel to the \(t\)-axis, so that the first transition in the chosen system of reference signifies rest, it is evident that

\[\int_1 \ d\tau = t, \ \int_2 \ d\tau < t\]

On this depends the retardation of the moving clock compared with the clock at rest. (94)

Evidently, Sommerfeld had already resolved the twin (clock) paradox in 1923 in essentially the same terms as I have given above.

What is crucial to this resolution is that the proper time calculated along a given path in spacetime is an invariant quantity: it retains the same value under transformation of inertial frame. It is for this reason that it “can claim the prerogative of representing the objective lapse of time”, to
use Gödel’s own words (558), and thus undermines his argument from the relativity of simultaneity to the unreality of time. Of course, Gödel assumed that an objective lapse would have to consist in a global plane of becoming, and therefore could not be relative to spacetime path; but, according to the point of view I am advocating here, this assumption is unwarranted in relativistic physics, where becoming is local, and dynamical change is parametrized by proper time, not co-ordinate time. It remains the case, of course, that the proper time is a maximum in the rest frame of an inertially moving object, and that in this circumstance it is numerically equal to the co-ordinate time. For when \( v = 0, \beta = 0, \) so that \( \gamma = (1 - \beta^2)^{-1/2} = 1, \) and \( t = t_0. \) But this is only numerical equality, not identity. It corresponds to the fact noted above that the longest time interval between two spacetime points in timelike separation is given by the straight line in spacetime connecting them. All other paths, whether the two inertial paths of the original Twin Paradox thought experiment, the paths of the Modified Twin Paradox incorporating a curve of deceleration and a spiral, or even a steady curve representing the travelling twin gradually slowing up turning round and retuming, are shorter. But by the same token, Special Relativity is perfectly able to account for these non-inertial paths, and for each of them the proper time would be calculated by an integration along the path in question, not by the difference in time co-ordinates in any inertial frame. If proper time were the time co-ordinate in an inertial frame at rest, \( t_0, \) it would not be applicable to such curved paths. In contrast, proper length is the length of an object—a metre stick, say—in a specific frame of reference, namely, the inertial frame in which it is at rest.

Still, it may be objected, proper length is nevertheless also an invariant quantity. Just as the length of a path joining two events in timelike separation is invariant under change of frame, so is the length of a curve joining two events in spacelike separation. Indeed, it is often argued that the analogy between it and proper time is perfect: “proper length is the invariant interval of a spacelike path whereas proper time is the invariant interval of a timelike path”.\(^ {16} \) Thus, it is suggested, the definition of proper length should be generalized so that it is the exact analogue of proper time: a line integral along a curve joining two spacelike separated events. But a little further reflection shows that this cannot be right: an arbitrary curve joining two spacelike separated events is not generally a length. It is only a length if all the points on the curve are simultaneous in some given reference frame. And while the path integral along such a curve is indeed independent of the

\(^ {16} \) Quoted from an article on proper length in Wikipedia (http://en.wikipedia.org/wiki/Proper_length; May 5, 2007). The author suggests a generalization of proper length so that it is given by the line integral \( L = c \int \sqrt{g_{\mu \nu} \, dx^\mu \, dx^\nu}, \) where \( g_{\mu \nu} \) is the metric tensor for the spacetime with +--- signature, normalized to return a time.
choice of reference frame, it has no particular physical significance. It does not even represent a path, in the normal acceptation of a path as a series of positions that can be successively traversed—as, for instance, by Harvey Brown’s waywiser (Brown 2005, front cover, p. 8)—for such an interval is timelike. Proper length is correctly defined as the path integral, not along an arbitrary curve joining the endpoints of the metre stick at the same time, but along the shortest curve, which is a straight line joining them in the frame at which they are at rest. If (elapsed) proper time were the strict analogue of this, it would be the longest time between two timelike separated events, which would be the time in a frame of reference at rest, i.e. the co-ordinate time. It is precisely this interpretation that I am attacking, Because proper length is the interval between two events at the same co-ordinate time, it is specific to a particular reference frame.

Thus proper time has a fundamentally different character from proper length. Although both are invariant under change of frame, proper length is the length of an object in its own rest frame, whereas proper time is independent of frame. In this respect proper length is analogous to proper mass. (It differs from the latter, however, in that proper mass seems to be an essential characteristic of an elementary body (such as an electron), whereas proper length is a contingent one.) At any rate, there is a fundamental dissymmetry between duration and length in Special Relativity, somewhat obscured by talk of their embodiments in observers’ clocks and rods. For whereas an observer’s clock measures proper time elapsed along a path, a dynamical variable specifiable independently of reference frame, the proper length of the observer’s measuring rod is specific to the inertial frame in which the observer is at rest. Thus we see that, ironically, there is a sense in which Minkowski’s introduction of proper time undermines his famous pronouncement at the beginning of his paper about the demise of time: “Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality”. (75)

I am by no means the first to point out the radical implications of relativity theory for our understanding of time. As Milič Čapek has stressed in several publications (1966, 1975, 1976), the invariance of Minkowski’s relations of being in the absolute past or future of an event means that in relativity theory the role of time is strengthened and made more distinct than in classical physics.

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17 Minkowski’s judgement is echoed by Einstein in his essay “The Problem of Space, Ether and the Field in Physics”: “Hitherto it had been silently assumed that the four-dimensional continuum of events could be split up into time and space in an objective manner... With the discovery of the relativity of simultaneity, space and time were merged in a single continuum...” (1954, 281-82).
The distinction between proper time and coordinate time is stressed by Larry Sklar in his treatment of the clock paradox;\(^\text{18}\) and Kent Peacock (1992), has also discussed the paradox in terms of a comparison between the proper times of the twins while they are spatially distant. But perhaps the clearest explanation of the distinction between “time co-ordinate” and “proper time” and its significance was given by Howard Stein:

Proper time is not a quantity attached to space-time points or to pairs of space-time points; it is in this respect a notion utterly different from the quantity “time” or “time interval” of pre-relativistic theory... The fundamental physical role of proper time comes from the principle (here stated roughly) that whenever a process takes place along a well-defined line of space-time (“world-line”), the time rates in the dynamical principles that govern that process are to be understood in terms of the proper time along that line (and not in terms of a “time coordinate”)...\(^\text{19}\)

Yet it seems to me that the significance of this degeneracy of time in relativity theory is still largely unrecognized. Philosophers and physicists continue to write as if it is the time co-ordinate function, or time in relation to an inertial observer, and not proper time, that measures the duration of processes in relativistic physics. This is implicit in all discussions that agree with Gödel in construing the objective lapse of time in terms of an infinity of layers of “now”, with these planes of simultaneity picked out by the time co-ordinate function in an inertial reference frame, such as the arguments of Putnam, Rietdijk and Maxwell discussed above.

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\(^{18}\) Sklar (1974, 268) correctly points out that, whereas “‘co-ordinate time’ between two events is relative to a given inertial frame”, “[p]roper time is defined only for events at timelike separations and only relative to a particular spacetime curve between the events. On the other hand it is an invariant notion.” Unfortunately, though, he goes on to claim that anyone who wishes to assert that future events are not real relative to the assertor is forced by the Putnam-Rietdijk argument to admit that such notions are “just as relative to an inertial state of motion of the assertor and just as ‘nontransitive across observers in different states of motion’ as we have made the simultaneity relation.” (275).

\(^{19}\) Stein (1968, 11, fn. 6). This quotation from Stein was my starting point in the line of argument for his paper. Cf. also p. 16: “… ‘a time co-ordinate’ is not ‘time.’ Neither a nor b is, in any physically significant sense, ‘present’ (or past) for any observer at c—regardless of his velocity—for neither has already become for c (nor has c for them); but a has already become for b, and can influence it.” [Here a and b are connectible by a time-like vector ab, the other pairs by space-like vectors ac and bc.]