Time and Inertia R. T. W. Arthur

Abstract: The status of time is very uncertain in contemporary physics, where it should, according to many theorists (e.g. Julian Barbour, Lee Smolin, and Carlo Rovelli), come out as an emergent or supervenient feature of the natural world. This motivated Barbour to go back to classical physics, and provide a Machian theory of Newtonian time in which, following the lead of Lange with his founding work of on reference frames, time in classical physics is implicitly determined by inertial motion with respect to all the other bodies in the universe. Here I present a different approach to the implicit determination of time by inertial motion, based on the group-theoretic derivation of the Galilean and Lorentz transformations by Jean-Pierre Provost.

In my interpretation, this derivation depends on Newton's idea of *relative* spaces, rather than the nineteenth century idea of reference frames infinitely extended in three spatial and one temporal directions. A *relative space* is characterized as a group of translations, where a translation is conceived as an inertial displacement in that space. Such a displacement, effected by a (point)-body undergoing inertial motion tracing out a straight line in Euclidean space, corresponds to a passive boost translation in the space. Inertial displacements are governed by Newton's equal times principle: "A body acted on by [two] forces acting jointly describes the parallelogram in the same time in which it would describe the sides if the forces were acting separately". The existence of such spaces is taken as given, rather than explained, so that the principle of inertia is the postulate that there exists an infinite class of equivalent relative spaces in relative motion one to another, forming a (differentiable and connected) one-parameter group. It is a theorem of group theory that such a group will have an additive parameter, here ζ, which (in order not to prejudge things) I denote by the neutral word "swiftness". The *relativity principle*, generalized from Newton's Corollary 5, is that any displacement x in any one inertial frame can be realized by an active boost translation of a point-particle at rest in a second inertial frame in motion relative to the first with a swiftness ζ. Apart from the standard assumptions of oddness of ζ. under space reflection, and a causality condition that ensures there exist time intervals that are invariant under transformations associated with any swiftness ζ, the only other assumption is one concerning time. According to Newton's equal times principle, two inertial displacements adding to a third must be effected in equal times. Therefore, according to the relativity principle, so must any two active boost displacements adding to a third, if they are successfully to realize the inertial ones. With these assumptions in place, Provost's derivation proves that Galilean and Lorentzian invariance are the only two possibilities for transformation groups.

In this derivation time is implicitly characterized by the equal times principle and the assumption that inertial displacements in a given relative space may be equivalently realized by active boosts of the relative space within which a particle is at rest. The times associated with different relative spaces then transform according to either the Galilean or the Lorentz transformations; with experiments of various kinds deciding in favour of the latter.