1. [**SL proof**] (a) *Prove the formal validity* of the following sequent:

   ¬(F & ¬G), ¬F → H, ¬H → ¬G ⊢ H

(1) ¬(F & ¬G)	Prem	
(2) ¬F → H	Prem	
(3) ¬H → ¬G	Prem	
│ (4) ¬H	Supp/RA	
│ (5) ¬G	3, 4 MP	
│ (6) ¬F	1, 5 CS	
│ (7) H	2, 6 MP	
│ (8) ⊥	4, 7 Conj	
(9) H	4-8 RA, DN	[4 marks]

2. [**SL proof**] (a) In the following "proof" of P → ¬O, ¬P → D ⊢ ¬P → (D v O) there are **2 *distinct mistakes*** *made in applying rules of inference* (as distinct from any strategic errors). *Identify the mistakes* (explaining them briefly)

(1) P → ¬O	Prem	
(2) ¬P → D	Prem	
│ (3) ¬P	Supp/CP	
│ (4) O	1, 3 MT —no, MT is *p*→ *q*, ¬*q* ⊢ ¬p! FDA!	
│ (5) D	2, 3 MP	
│ (6) D v O	5, 4 Disj—no, should be 5 Disj!	
(7) ¬P → (D v O)	3-6 CP	[4 marks]

(b) ***give a correct proof*** *of the formal validity of the sequent in* (b).

(1) P → ¬O	Prem
(2) ¬P → D	Prem
│ (3) ¬P	Supp/CP
│ (4) D	2, 3 MP
│ (5) D v O	4 Disj
(6) ¬P → (D v O)	3-6 CP

[6 marks]

Continued on next page…

3. [**SL symb^{n}, proof**] *(a)* ***Symbolize*** *the following argument* given by physicist Lee Smolin, *and (b)* ***give a proof of its validity***:

E → ¬C, N → B, B → C, E ∨ N ∴ C ↔ B [4 marks]

(1) E → ¬C	Prem
(2) N → B	Prem
(3) B → C	Prem
(4) E ∨ N	Prem
	(5) C	Supp/CP
	(6) ¬E	1, 5 DN, MT
	(7) N	4, 6 DS
	(8) B	2, 7 MP
(9) C → B	5-8 CP
(10) C ↔ B	9, 3 Conj, BI

[4 marks]

4. [**SL T-trees**] *Using* ***the method of truth trees*** *determine whether each of the following sequents is valid or invalid*:

(a) (D & O) → ¬I, D ⊢ I → ¬O

1. √ (D & O) → ¬I — Prem
2. √ D — Prem
3. √ ¬(I → ¬O) — ¬Concl
4. I — 3 ¬→
5. √ ¬¬O
6. O — 5 ¬¬
7. √ ¬(D & O) | ¬I ⊥ — 1→
8. ¬D ⊥ | ¬O ⊥ — 7¬&

The tree is **closed** (and also therefore **complete**); the sequent is therefore formally VALID. [4 marks]

Continued on next page…

(b) B ↔ (C & D), ¬C ⊢ ¬D v B

1.	√	B ↔ (C & D)		Prem
2.		¬C		Prem
3.	√	¬(¬D v B)		¬Concl
4.	√	¬¬D		3 ¬v
5.		¬B		
6.		D		4 ¬¬
7.	B		¬B	1↔
8.	⊥	√	¬(C & D)	
9.				
10.		¬C	¬D ⊥	8¬&

The tree is **complete**, but still has a complete open path (with literals ¬C, ¬B and D); the sequent is therefore formally INVALID.

[4 marks]

5. [**SL T-table**] ***Determine*** *using a* ***truth table*** *whether the following abstract statement is a* ***contradiction, a tautology or a contingent statement:***

[F & (¬G → G)] ↔ (F v ¬G)

F	G	[F	&	(¬G	→	G)]	↔	(F	v	¬ G)
T	T	T	T	F	T	T	T	T	T	F
F	T	F	F	F	T	T	T	F	F	F
T	F	T	F	T	F	F	F	T	T	T
F	F	F	F	T	F	F	F	F	T	T
		(1)	(5)	(2)	(4)	(1)	(6)	(1)	(3)	(2)

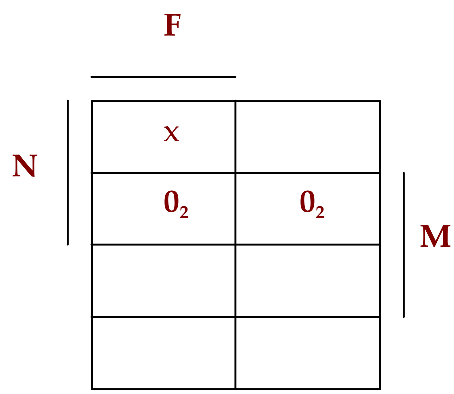
This is a *contingent statement*, as the column (6) shows.

[4 marks]

Continued on next page…

6. [**Carroll diagrams**] ***Determine whether each*** of the arguments in (a) and (b) ***is valid*** or ***invalid*** using the ***Carroll diagram method*** (with a brief explanation of your answer):

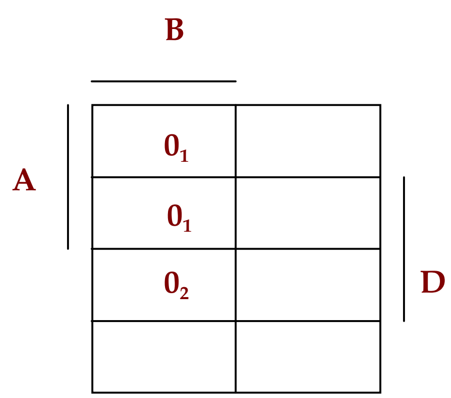
(a) Some NEUTRINOS go FASTER than light. But no neutrinos have rest MASS. It follows that some things with no rest mass go faster than light.



Justify your answer here: The things in cell 1 are things with no rest mass (not-M) that go faster than light (F), so the argument is therefore **valid**.

[3 marks]

(b) ARIANS do not BELIEVE in Christ's divinity. Therefore Arians are DEISTS, since no Deists believe in Christ's divinity.



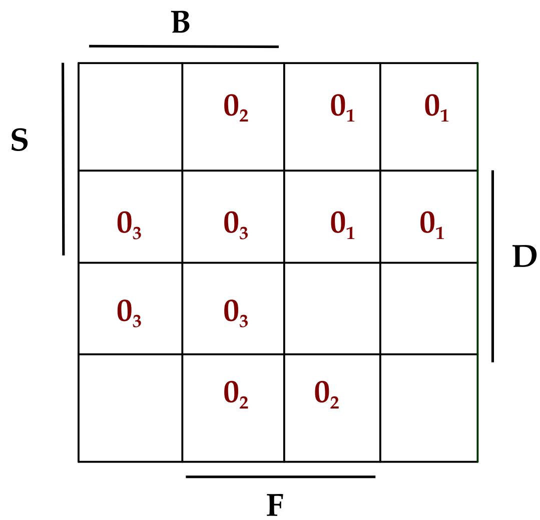
Justify your answer here: "Arians are DEISTS" is the conclusion, but it cannot be read off the diagram, since it is possible that there are Arians who are not Deists (cell 2). Therefore **invalid**.

[3 marks]

Continued on next page…

(c) *Assuming that every predicate is to appear exactly twice in the argument, use a* ***Carroll diagram*** *to work out* ***what conclusion*** *(express it in colloquial English!)* ***may be validly inferred*** *for the following 4-predicate argument*:

All of the SULTAN'S camels are BACTRIAN. Only DROMEDARIES move FAST. No Bactrian camel is a Dromedary. [UD: camels]



Predicates used only once in the premises: S, F. But the S-F overlap is empty. Therefore the conclusion is

None of the Sultan's camels are fast. Or, equivalently,

No fast-moving camels belong to the Sultan.

[4 marks]

7. [**PL symb**^{n}] ***Symbolize the following statements*** of ***predicate logic*** using the suggested abbreviations:

(a) There are CALIFORNIANS who do not SURF.

$\exists x(Cx \,\&\, \neg Sx)$

(b) VEGETARIANS don't eat MEAT.

$\forall x(Vx \rightarrow \neg Mx)$

(c) Assad is a DICTATOR who FORFEITED the right to rule.

$Da \,\&\, Fa$

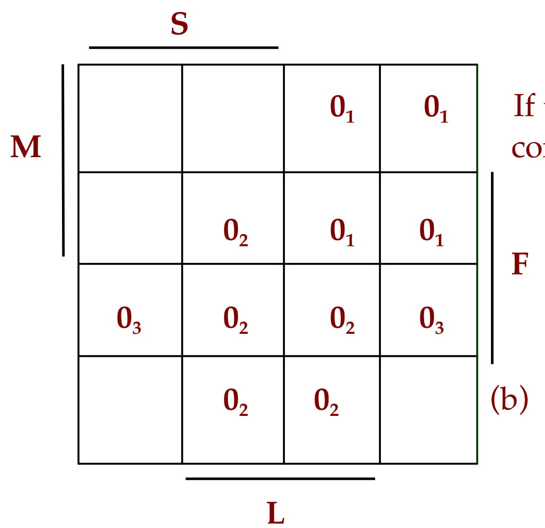
(d) Every STATEMENT is either TRUE or not PROVEN.

$\forall x[Sx \rightarrow (Tx \vee \neg Px)]$ [1 mark each]

Continued on next page…

8. [**PL symb^n + proof**] The following argument is **penevalid**. (a) *Determine the one-predicate existential premise that would render it valid,* (b) *symbolize the argument together with this premise,* and (c) *prove its validity using predicate logic.*

All MICRO-organisms are too SMALL to see. No FUNGI are LEMONS. But all fungi are micro-organisms. It must then follow that some things that are too small to see are not lemons.



If there are Fs, cell 5 gets an x, and the conclusion follows. So, (a) ∃x Fx

[2 marks]

(b)

(b) ∀x(Mx → Sx), ∀x(Fx → ¬Lx), ∀x(Fx → Mx), ∃x Fx ∴ ∃x(Sx & ¬Lx)

(c) (1) ∀x(Mx → Sx) Prem [4 marks]

(2) ∀x(Fx → ¬Lx)	Prem
(3) ∀x(Fx → Mx)	Prem
(4) ∃x Fx	Prem (Implicit)
(5) Fi	4 EI
(6) Fi → Mi	3 UI
(7) Fi → ¬Li	2 UI
(8) Mi → Si	1 UI
(9) Mi	5, 6 MP
(10) Si	8, 9 MP
(11) ¬Li	5, 7 MP
(12) Si & ¬Li	10, 11 Conj
(13) ∃x(Sx & ¬Lx)	12 EG

[4 marks]

Continued on next page…

9. **[PL proofs]** *Prove the **validity** of the following sequent:*

Pf, ∀x(Px → Mx) ⊢ ∃x(Mx & Px) [6 marks]

(1) Pf	Prem
(2) ∀x(Px → Mx)	Prem
(3) Pf → Mf	2 UI
(4) Mf	1, 3 MP
(5) Mf & Pf	4, 1 Conj
(6) ∃x(Mx & Px)	5 EG

10. **[PL proof]** In the following ***incorrect proof*** *of* ∀x(Fx → Gx) ⊢ ∀x¬Gx → ∃x¬Fx, there are **2 *distinct mistakes*** *made in applying rules of inference or use of arbitrary names*. (a) ***Identify them*** (explaining them briefly), and

(1) ∀x (Fx → Gx)	Prem
(2) ¬Gu	Supp/CP
(3) Fu → Gu	1 UI
(4) ¬Fu	2, 3 MT
(5) ∀x¬Gx	2 UG –violates first restriction on UG: u in
(6) ∃x¬Fx	4 EG undischarged supposition
(7) ∀x¬Gx → ∃x¬Fx	5-6 CP–antecedent of conditional in CP must be the Supp [4 marks]

(b) ***give a correct proof of validity*** of the above sequent.

(1) ∀x (Fx → Gx)	Prem
(2) ∀x¬Gx	Supp/CP
(3) ¬Gu	2 UI
(4) Fu → Gu	1 UI
(5) ¬Fu	3, 4 MT
(6) ∃x¬Fx	5 EG
(7) ∀x¬Gx → ∃x¬Fx	2-6 CP [6 marks]

Continued on next page...

11. [**RL symb**^{n}] *Using the suggested notation, **translate** the **relational** statements:*

(a) Joyce is $TALLER_2$ than Mario.

jTm

(b) Someone is $TALLER_2$ than Mario.

∃x xTm

(c) All the WOMEN are $TALLER_2$ than Mario.

∀x (Wx → xTm)

(d) Some of the WOMEN are $TALLER_2$ than any of the MEN.

∃x [(Wx & ∀y(Mx → xTy)] [1 mark each]

12. [**RL proof**] (a) ***Prove the validity*** *of the following sequent in relational logic:*

∀x ¬xRx , ∀x∀y∀z{(xRy & yRz) → xRz} ⊢ ∀x∀y(xRy → ¬yRx)

(1) ∀x ¬xRx	Prem
(2) ∀x∀y∀z{(xRy & yRz) → xRz}	Prem
(3) (uRv & vRu) → uRu	2 UI
(4) ¬uRu	1 UI
(5) ¬(uRv & vRu)	3, 4 MT
(6) ¬uRv ∨ ¬vRu	5 DM
(7) uRv→ ¬vRu	6 MI
(8) ∀x∀y(xRy → ¬yRx)	7 UG
or	
│ (6) uRv	Supp/CP
│ (7) ¬vRu	5, 6 CS
(8) uRv→ ¬vRu	6–7 CP
(9) ∀x∀y(xRy → ¬yRx)	8 UG

[5 marks]

(b) ***What properties*** *of the relation* R *are represented by the premises and conclusion respectively?*

∀x ¬xRx	—irreflexivity
∀x∀y∀z{(xRy & yRz) → xRz}	—transitivity
∀x∀y(xRy → ¬yRx)	—asymmetry

[3 marks]

Continued on next page…

13. [**wffs**] (a) *Two of the following formulas are **not** wffs. **Identify which two***:

(i) ¬(¬R) v S (ii) ∃x∀y(Ax → Dy) (iii) ∃xAx → ∀y(Bx v Sy) (iv) Am ∨ Jr

(i) and (iii) are not wffs. [2 marks]

(b) *For the two that **are** wffs, **explain briefly** how each of them can be constructed by the rules of wff-formation.*

Am and Dn are wffs by clause (i), (Am → Dn) by clause (iii), ∀y(Am → Dy) by clause (iv), and ∃x∀y(Ax → Dy) by clause (iv) again.

Am and Jr are wffs by clause (i), Am ∨ Jr by clause (iii) and convention about outermost groupers. [2 marks]

(c) ***Identify the scope of the existential quantifier in*** (ii) ***and*** (iii).

For (ii) it is ∀y(Ax → Dy)

For (iii) it is Ax [2 marks]

14. [**SL, PL**] The following argument is an instance of a fallacy in ***statement logic***:

(a) If *p* then *q*, ¬*p* ∴ ¬*q*. This is the fallacy of *denying the antecedent*, FDA [2 marks]

(b) ∃x(Bx & Px) → Pv, ¬∃x(Bx & Px), Bv ∴ ¬Pv [2 marks]

(c)

(1) ∃x(Bx & Px) → Pv	Prem
(2) ¬∃x(Bx & Px)	Prem
(3) Bv	Impl. Prem
(4) ∀x¬(Bx & Px)	2 QN
(5) ¬(Bv & Pv)	4 UI
(6) ¬Pv	3, 5 CS

[2 marks]

(d) An argument is ***formally valid*** if it is an instance of a valid form, as this is; even if it is also an instance of an ***invalid*** form, as this is. So the argument is formally valid. (It is also Chrysippian-valid, as the conclusion cannot be denied if the premises are accepted.) [2 marks]

THE END