Problem Set 1 Origins 3A03

1. Huygens corrected Descartes's law that the total "quantity of motion" (roughly, size x speed) is conserved in every collision of bodies, by showing that it should be $\sum_i m_i \underline{v}_i$ (the total *momentum*, as we now call it) that is conserved, where m is the mass and $\underline{v}_i$ is the *directed velocity* (i.e. a vector quantity, not a scalar speed). But, Huygens insisted, the velocity in question is a *relative velocity*.

Problem: On a canal boat, a body of mass M and velocity V collides with a stationary body of mass m, and they then move off stuck together with velocity v in the same direction. (All these velocities are with respect to the boat.) With respect to the bank of the canal, the boat is moving at a velocity $-V$, so from there before the collision the mass M appears to be at rest while the smaller mass m moves with velocity $-V$; after the collision the two masses move off together with velocity w with respect to the bank.

(a) calculate the total momentum before and after the collision (i) with respect to the boat, and (ii) with respect to the bank.

(b) show that if the total momentum is conserved in all collisions, then the combined bodies move with respect to the bank at a velocity $w = v - V$, consistently with Huygens' claim that it is relative velocity that is conserved.

2. Isaac Newton claimed that he had established already in 1666 that the Moon might be held in orbit by the same force responsible for an apple's falling to the ground. Moreover, he claimed that he had already deduced the inverse square law by combining Kepler's third law ($T^2 = kr^3$) with the formula he had by that time derived for the centrifugal force, $F = mv^2/r$, assuming that the Moon's orbit is circular, and that the Moon is kept in orbit by the balance between the centrifugal force and its attraction to the Earth.

(a) Show that on these assumptions (*and without making any other assumptions*) it does indeed follow that the force of attraction on the Moon is proportional to the inverse square of its distance from the centre of the Earth. **NB: do NOT use any of Newton's laws, such as F = ma. Newton had not formulated them yet!**

(b) Show further that if the Moon is 60 times further from the centre of the Earth than an apple at its surface, and the Moon falls 0.0044 feet per second per second, as Newton estimated, the assumption that the same attractive force is acting on both apple and Moon gives a result for the acceleration of the apple at the Earth's surface that "agrees pretty nearly" with the accepted value of around 16 feet per second per second.