Time Lapse and the Degeneracy of Time:

Gödel, Proper Time and Becoming in Relativity Theory

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1. Preview of the argument

In the transition to Einstein's theory of Special Relativity (SR), certain concepts that had previously been thought to be univocal or absolute properties of systems turn out not to be. For instance, mass bifurcates into (i) the relativistically invariant *proper mass* m_0, and (ii) the mass relative to an inertial frame in which it is moving at a speed $v = \beta c$, its *relative mass* m, whose quantity is a factor $\gamma = (1 - \beta^2)^{-1/2}$ times the proper mass, $m = \gamma m_0$.

By an extension of a term already used in physics, I call this phenomenon *degeneracy*: Just as an energy state is considered degenerate if it is in fact a multiplicity of energy states that are not distinguishable from one another until, say, a magnetic field is applied, so too it turns out that the concept of mass is degenerate: it is only at speeds that are an appreciable fraction of the speed of light that the two different concepts of mass are distinguishable. In the same way one can say that Special Relativity shows that the concept of time is likewise degenerate:

(i) there is the *relative time (or time co-ordinate function) t*, whose quantity varies (like relative mass) according to the inertial frame chosen, and

(ii) there is the *proper time* τ which is invariant under change of frame, and calculated by an integration along the path taken.

This much is well known. But what I think has been insufficiently appreciated is the change in the ontology of time accompanying this degeneracy, especially with respect to issues concerning becoming and process. For the classical concept of time does double duty in

(a) correlating distant events as prior to, simultaneous with, or after, some given event; and

(b) measuring or determining how fast things age, that is how fast the properties of a given system change, or how fast the states of a given process follow one another.

But whereas classically it was assumed that events occur (or states successively come into existence) in the present, where each successive present is assumed to be a unique worldwide instant, this conception is untenable in relativistic physics because of the relativity of simultaneity. That is, if one accepts that the becoming of an event at a particular point in spacetime should be invariant, yet construes its becoming in terms of coming to be simultaneous with a given observer, then one can derive a contradiction. This circumstance has persuaded many physicists and philosophers, pre-eminent among them Kurt Gödel, that one must abandon the idea of becoming altogether. I shall argue that such arguments are fallacious precisely because they keep fused together the two distinct functions of time that are (degenerately) fused in the classical conception of time: (a) determining a plane of simultaneity, and (b) invariant time lapse. In SR the becoming of events in succession, the rate of a process or the rate at which a thing ages, is tracked by *proper time*; the synchronicity of distant events is tracked by the *time-coordinate function*. This separation of these two different aspects of time into two different time concepts τ and t is characteristic not only of SR, but of all relativistic physics, where every timelike curve represents a possible process, whose rate of evolution is parametrized by proper time.

This connection of becoming or process with proper time as opposed to co-ordinate time is masked, so I argue, by the way many physicists and philosophers talk about proper time as if it is simply the *relative time of an observer in his or her own rest frame*. This is particularly evident in some discussions of the Twin Paradox of SR. By paying careful attention to what the twins could actually observe and infer about each other's times, I argue that it is implicit in the correct resolution of the paradox that time lapse is correctly measured by proper time, and show how, *contra* Gödel, this involves a path-dependent and local notion of becoming. I show how, despite the bifurcation of length in relativistic contexts into frame-dependent and proper length concepts, this has no parallel ontological significance, and briefly discuss this disanalogy between proper time and proper length. In closing, I argue that the detachment of objective becoming from the issue of the existence of invariant time slices undercuts Gödel's arguments for the ideality of time, and discuss the implications of the fact that a timelike curve must represent the path of a possible process.[1]

2. Gödel's Arguments for the Unreality of Time

In 1949 Gödel published his famous solution to the Einstein Field Equations for the General Theory of Relativity, representing a rotating universe in which there exist closed time-like curves (1949a). That is, in such worlds for any two points P and Q on a worldline of matter occurring in the solution, with P preceding Q on this line, "there exists a [continuously future-directed] timelike line connecting P and Q on which Q precedes P; i.e. it is theoretically possible in these worlds to travel into the past, or otherwise influence the past" (447). As is well known, in the same year he published a second paper drawing out the philosophical implications of this, and promoting an idealist philosophy of time(1949b). Gödel argued that for every assignment one might make of a cosmic time function for such models of the universe, "one could travel into regions of the universe which are past according to that definition," thus showing that "an objective time lapse would lose every justification in these worlds" (561). This being so, the assumption of an objective time lapse should be abandoned altogether: time, as Parmenides and Kant had argued, is bound up with our particular way of perceiving the world, and is not an objective feature of the four-dimensional totality.

Less remarked upon is the argument Gödel gives for the same idealistic conclusion earlier in that paper, based on the Special Theory of Relativity. He argues that the assertion that two spatially separated events *A* and *B* are simultaneous "loses its objective meaning, insofar as another observer, with the same claim to correctness, can assert that *A* and *B* are not

[1] A complementary analysis of Gödel's arguments, from very much the same point of view as that offered here, is given by Dennis Dieks in his (2006).

simultaneous (or that *B* happened before *A*)" (557). This, he claims, allows one to construct "an unequivocal proof for the view of those idealistic philosophers who, like Parmenides, Kant and the modern idealists, deny the objectivity of change and consider change as an illusion or an appearance due to our special mode of perception" (557). The proof he gives runs as follows:

> Change becomes possible only through the lapse of time. The existence of an objective lapse of time, however, means (or at least is equivalent to the fact) that reality consists in an infinity of layers of "now" which come into existence successively. But, if simultaneity is something relative in the sense just explained, reality cannot be split up into such layers in an objectively determined way. Each observer has his own set of "nows", and none of these various systems of layers can claim the prerogative of representing the objective lapse of time. (557-8)

Gödel's idealistic conclusion is of course a radical one, and few modern philosophers of science accept it. According to the dominant view in the philosophy of science in the latter half of the last century, time intervals as measured in the various possible frames of reference are all perfectly objective, even if they are not invariant. Relativity of the duration of a process, it is argued, no more entails its subjectivity or illusory nature than relativity of the mass of a system to frame of reference entails the subjectivity or illusory nature of mass. The relativity of simultaneity entails that no one of these relative times can be privileged as the "actual time", just as Gödel had argued. Nevertheless, each measure of duration is consistently related to any of the others by the Lorentz transformation formulas. According to the dominant view—as subscribed to, for instance, by Jack Smart, Adolf Grünbaum, Paul Davies, and many others[2]—what is refuted by such arguments from the relativity of simultaneity is not the objectivity of time lapse, but the notion of *coming into existence*. It is true, as Gödel observed, that different

[2] See Smart (1968, 255ff.) and (1980); Grünbaum (1976); Davies (1989, 3): "Thus relativity physics has shifted the moving present out from the superstructure of the universe, into the minds of human beings, where it belongs… present day physics makes no provision whatever for a flowing time…".

choices of inertial reference frame will result in wholly different classes of events being simultaneous with a given event, and that one must therefore relinquish the classical notion of a world-wide "now". But what this precludes is not the objectivity of time lapse, but —as Hilary Putnam (1967) and C. W. Rietdijk (1966) each argued on grounds similar to Gödel's in the late 60's— any notion of *objective becoming (becoming real*, in Putnam's case, *becoming determined* in Rietdijk's*)*. (A similar argument was given by Nicholas Maxwell (1985).) Thus although time lapse is perfectly objective, it is frame-dependent.

Interestingly, Gödel, anticipated this objection that the relativity of time lapse "does not exclude that it is something objective." To this he countered that the lapse of time connotes "a change in the existing", and "the concept of existence cannot be relativized without destroying its meaning completely" (558, n. 5). The dominant view, by contrast, would urge that the relativity of existence is avoided precisely by denying that time lapse constitutes a "change in the existing": the existence of events is their existence in a four-dimensional spacetime, and this does not change. Against this, I have argued elsewhere (Arthur 2006, 131-136) that the sense in which spacetime "exists" is not a temporal sense, and so will not support the contention of Putnam *et al*. that events simultaneous with another event are "already real" for it; to suppose that this is so, I argue, leads inexorably to a conclusion that denies the reality of temporal succession.[3]

What I wish to draw attention to here, however, is a premise that the dominant view shares with Gödel's: both assume that events are real or determined when they are present to an observer, with presentness construed in terms of simultaneity in the observer's frame of reference; i.e. they construe the reality of an event *in terms of the time co-ordinate function*. Thus Putnam (1967) and Rietdijk (1966) each assume that becoming real or determined must

[3] For a thorough analysis of the problematic notion of *existence* in a temporal context see Steve Savitt (2006) and Mauro Dorato's article (2006) in the same volume.

occur relative to "an observer's inertial system", with time-lapse measured by the time co-ordinate function, as a premise in their *reductio* arguments against the reality of becoming real or determinate. The crucial premise here is the Gödelian one that for each individual observer, "the existence of an objective lapse of time … is equivalent to the fact that reality consists in an infinity of layers of 'now' which come into existence successively." That is, the time lapse between, for instance, two events in anyone's life history is given by the difference in the values of the time co-ordinate function in some particular inertial reference frame.

But this construal of time lapse in SR is false, as can be shown by an analysis of the much discussed Twin Paradox. Here we imagine one twin staying at home while the other speeds off at a relative velocity which is an appreciable fraction of *c*, the speed of light, turns round, and returns at a similar velocity. When they are reunited, less time has elapsed for the travelling twin, who is consequently found to have aged less. But the discrepancy between the times elapsed for the two twins cannot be a discrepancy between times as measured by *co-ordinate time* —the time or "layer of 'now'" associated with some given inertial system— since in that inertial frame of reference the twins are apart for exactly the same time, as measured by the time co-ordinate of that frame. Indeed, in any such inertial frame, there is only one difference between the co-ordinates of these two points, and not one for each twin. In fact, the time taken for the twins to make each of their trips through spacetime from the point at which the travelling twin departed to the later point of their reunion must instead be determined by integrating the *proper time* along each twin's particular world line. Thus the root of the trouble with the "layer of now'" conception of time lapse is a failure to take into account the degeneracy of time. Time lapse is measured by the proper time. The difference in the proper times for their journeys is not the same as the difference in the time co-ordinates of the two points in some inertial reference frame. If time lapse were measured by such a time co-ordinate function, then both twins would be the same age. They are not. *Ergo*, time lapse (in the sense of how long a given process takes, how quickly it becomes) is not measured by the time co-ordinate function. So Gödel's "unequivocal proof" of the ideality of time falls flat on its face.[4]

It is puzzling that this simple consideration is not widely recognized; this suggests that there are other assumptions at work that mask its application. I believe they have to do with a *misconception of proper time as the time co-ordinate of the observer's rest frame*, and related misconceptions about *an observer* "*inhabiting an inertial frame*" and "*experiencing*" the events which are simultaneous with his or her state of consciousness. Rietdijk, for example talks of two spatially separated observers "experiencing the same present ... in virtually the same inertial system" (1966, 342), Grünbaum writes that for an organism *M* to experience an event at a time *t* is to be "*conceptually aware* of experiencing at that time either the event itself or another event simultaneous with it in *M*'s reference frame" (1976, 479), and Putnam of "everything that is simultaneous to you-now in *your* co-ordinate system" being real, and Clifton and Hogarth of two observers' "inhabit[ing] the same inertial frame" (1995, 379). Although misconceptions about proper time are seldom stated explicitly, they also appear to be quite prevalent. Indeed, they afflict the understanding of SR itself, as witnessed by some of the attempted resolutions of the Twin Paradox.

These considerations motivate another look at the Twin Paradox, to get clear on what is in an observer's (visual) experience in a relativistic context, and what is inferred; and to see more clearly how the distinction between proper time and co-ordinate time cleanly resolves the paradox without reference to the events one "experiences" as present undergoing a dramatic change, (Section 3) or implying that the discrepancy in the twin's ages is a General Relativistic

[4] It may be countered, as by my anonymous referee, that Gödel's argument depends only on the lapses of time being different for any two arbitrary curves connecting two timelike related events, and that Gödel does not assume that time lapse is measured by a time-coordinate function. But Gödel explicitly construes time lapse in terms of co-ordinate time in his argument from Special Relativity, where his argument against the "relativization of existence" crucially depends on this. This is supported by the interpretation of Palle Yourgrau (1991), who construes Gödel's argument as depending on a conception of time lapse as relative to reference frame.

effect (Section 4). This may seem otiose, given the number of times the paradox has been resolved, and given that no one who knows relativity thinks it a problem. But anyone who believes that the resolution of the paradox requires General Relativity, or a recognition that the events experienced as present by the moving twin undergo a discontinuous shift at the point of return, or that there is any asymmetry in what speeding up or slowing down of clocks is seen by or inferred about either twin of the other, has not properly appreciated, so I would claim, the profundity of the changes in our understanding of time wrought by Special Relativity.

3. The Twin Paradox revisited

To make our well-worn paradox vivid, let us take Terence the true Tellurian, who tethers himself to terra firma; and Astrid the astronaut, who abridges her age by abandoning Earth with alacrity for Alpha Centauri. (Since the tale of the two twins has been told so many times, I hope I may be allowed a little alliteration in the account.) We'll assume, to keep the figures round, that Alpha Centauri is 6 light years away, and that Astrid approaches it at six tenths of the speed of light (.6*c*), turns round in an instant, and returns towards Earth at the same speed (all of this in Terence's rest frame, i.e. from the point of view of an inertial frame of reference in which Terence is stationary). An easy calculation shows that, according to Terence, his twin is away for exactly twenty years (ignoring for now any periods of acceleration or deceleration). Things are otherwise for Astrid. At such great speed, distances are foreshortened by a factor $\sqrt{(1 - 0.36)} = \sqrt{(0.64)} = .8$: it appears to her that she makes a journey outwards of only 4.8 light years, and does it in 8 years. (Seen from Terence's perspective, time runs more slowly on her watch.) An equal space contraction and time dilation occurs on her way home, so that she takes only 16 years for her journey (still discounting the time of her deceleration). Thus when they are reunited and compare their watches, they find that Terence has aged 4 years more than his sister! This seems obviously paradoxical. If all inertial motion is relative, how can there be an absolute difference in their lifetimes resulting from it? The paradox is heightened by this observation:

while the twins are in relative inertial motion, each's duration will be running slower from the perspective of the other's rest frame. In each leg of the journey, Astrid would infer processes to be happening more slowly on Earth as it receded from her or approached her at 0.6*c*: her eight years would correspond to an inferred duration of processes on earth of only 6.4 years! As Herbert Dingle reasoned,[5] if each twin's life-process is slowed down relative to the other, each will age less than the other, an obvious impossibility! This is the paradox.

A standard resolution given of this paradox explains that the reason for the discrepancy is that as it is only Astrid who undergoes an acceleration as she turns around, it is therefore she who performs an absolute motion, not Terence. On this analysis, so long as the twins are in inertial motion relative to one another, each twin must indeed infer that the other's clock is running slow. The reason for the discrepancy is that, as Astrid turns about, the act of her deceleration skews her temporal orientation violently, and under the conditions stated, instantaneously. As she journeys home she infers Terence's clock to advance only 6.4 years, yet it will read 20 when she returns to Earth. So, at the time she sets out from Alpha Centauri for the journey home, it must read 13.6. Yet the instant before, the instant she arrives at Alpha Centauri, she would have inferred it to have read 6.4 years! So, instantaneously, it would have had to have jumped 7.2 (= 13.6 – 6.4) years.

So it's not that Astrid's instantaneous (and wholly unphysical) acceleration introduces a time dilation; it's that it discontinuously skews her temporal orientation. Now if we were to follow Putnam's informal way of speaking, we would say that Astrid "experiences" 7.2 years going by in an instant: events that were "present according to her co-ordinate system" are discontinuously displaced 7.2 years into the past of "her-now" according to that same system. In actual fact, however, no such wrenching change of her experience of the present occurs. These

[5] Herbert Dingle (1890-1978) was an English astronomer who wrote a standard textbook on relativity theory before becoming a vociferous opponent in his old age. See his 1972.

are facets of a sloppy use of the ideas of “observer’s reference frame” and the observer’s “present”, and a failure to distinguish between the time an observer might *infer* an event to occur from when the observer would *see* it occurring.[6] In fact, it will be worth going over the whole thing in some detail to see how this could be the case. Taking my cue from the lucid account of the twin paradox given by Paul Davies in his recent book, I shall re-examine the thought experiment by conducting it in three stages.[7]

First, just to get our bearings, let us assume a Cartesian cosmos, in which light is a *pression* that is transmitted instantaneously, and durations are completely independent of the state of motion of the enduring thing. We equip each twin with a very powerful telescope and a very large clock, and then suppose Astrid to leave for Alpha Centauri at 0.6*c*. This is very straightforward. The Tellurian twin sees his astronaut sister arriving at Alpha Centauri 10 years after she left, and sees his sister’s clock register those 10 years. When Astrid arrives at Alpha Centauri she sees Terence’s clock register that ten years have passed, and sees him aging a year per year as she returns home.

Now let’s assume a Dopplerian universe. In this universe, it is known that Descartes was wrong not to have listened to his mentor Beeckman: light travels at a finite speed *c* (in Terence’s Tellurian frame of reference). Otherwise the universe is classical, as before. Now things are interestingly different for the interstellar twins: because light takes 6 years to travel from Alpha Centauri to Earth, when Terence actually sees the event of Astrid’s arrival there, 16 years will have passed since they parted! He sees Astrid’s clock register only 10 years while his has registered 16. (Astrid’s clock is apparently running slow by a factor 5/8 compared to his). He is then even more surprised to see his sister take only 4 years to return, and watches his sister’s

[6] Cf. Lawrence Sklar (1974, 272): “One must always be careful in special relativity to distinguish what an observer actually *sees*, literally, from what he computes to be the case”.
[7] Davies (1995, 59-65) considers the twin paradox by “equip[ping] our twins with a powerful telescope so that they can watch each other’s clocks throughout the journey,” and then discusses what clock readings they would see, distinguishing the Doppler effect from the time-dilation effect.

clock running at 2.5 times the speed of his own; thus, as Terence views Astrid's return trip and all the processes happening in it, he sees them appearing to occur four times as fast (2.5 ÷ 5/8) as during the trip outwards! Astrid has an analogous experience. When she arrives at Alpha Centauri, she observes Terence's clock to be reading only 4 years. For the image of the clock registering 4 years travels the 6 light-years to Alpha Centauri to arrive there 10 years later (all from Terence's frame of reference). So Astrid sees Terence's clock has been running 2.5 times as slowly as hers (i.e. at 2/5 speed)! But on the way home she sees it to be running 1.6 times as fast: in the ten years it takes Astrid to return, Terence appears to her to age 16 years! Thus she, too, is puzzled to see her twin's clock going 4 times as fast (1.6 ÷ 2/5) as it was on the outgoing leg of the journey. But the twins' puzzlement is relieved when they learn about the Doppler effect: Events and processes occurring in a frame of reference in motion towards an observer appear to be speeded up ("blue shift"); occurring in a frame of reference in motion away from an observer they appear to be slowed down ("red shift").

Still, this does not explain the discrepancy in their experiences. Granted there is a certain symmetry: each twin sees the other's clock running 4 times as fast on the return trip as it was on the way to Alpha Centauri. But if all inertial motion is relative, they should have experienced the same red shift while moving apart, and the same blue shift when moving back towards one another. That they didn't is explained by the fact that we have taken the speed of light to be c in Terence's frame only. If the speed of light is also c in Astrid's frame of reference, then the situations are entirely symmetrical, and Astrid should see Terence's clock run slow on the outward trip by a factor of 1.6, and fast on the return journey by a factor of 2.5. Thus when she reaches Alpha Centauri, she should see Terence's clock read 10 × 5/8 = 6.25 years. But then Terence would age 20 – 6.25 = 13.75 years while she returns. This is a long way short of his aging calculated by the Doppler factor 2.5, 10 × 2.5 = 25 years, an obvious impossibility! In short, the assumption that the speed of light is the same in all frames of reference is at variance with the assumption of classical physics that all inertial motion is relative.

This, then, is the kind of discrepancy that physicists faced at the beginning of the twentieth century. All the experimental evidence seemed to suggest that the speed of light is the same in all inertial reference frames. But this is incompatible with the requirement that everything will appear the same from each inertial reference frame, unless something else gives. In terms of the twin example, the only way for Astrid to see a Doppler effect equal to Terence's is if the length of the journey in a frame of reference in relative motion were to suffer a contraction along the direction of motion by a factor of $\sqrt{(1 - v^2/c^2)} = \sqrt{(1 - 0.36)} = 0.8$. (That the dimensions of a body are distorted in this way by their motion through the aether was independently suggested by George FitzGerald and Hendrik Lorentz,[8] taken up by Joseph Larmor, and generalized by Henri Poincaré.) But this also implies a similar effect on the rates at which processes occur: they must slow up by the same factor in a frame of reference in relative motion. In terms of the present example, if the twin in motion at 0.6*c* covered a distance of only 4.8 light years (6 × 0.8) in her own frame of reference, then this would take her only 4.8 ÷ 0.6 = 8 years (= 10y × 0.8) in that frame. Time for the moving twin would, from the point of view of the stationary one, run more slowly by a factor of 0.8. This is the so-called *time dilation* effect, partially understood by Larmor and Lorentz, but first explicitly articulated by Einstein,[9] and now known to be a really occurring effect. Lorentz and company, of course, assumed that all these dilations and contractions could be referred to the frame in which the aether is at rest, and would effectively prevent one from detecting which frame this is. This is where Einstein stepped in: he dispensed

[8] Harvey Brown, in his (2005, 3, 45-55), explains that the conjecture of FitzGerald and Lorentz was that the dimensions of the body were altered in a certain ratio, not that there was a physical contraction along one of them. See also Mauro Donato, "Relativity theory between structural and dynamical explanations," forthcoming in *International Studies in Philosophy of Science*, preprint p. 7.

[9] For an authoritative discussion of the extent to which Larmor, Lorentz, and Poincaré did and did not anticipate Einstein's discovery of time dilation, see Brown (2005), esp. ch. 4.

with the unobservable-in-principle aether, so that each inertial frame would be on the same ontological footing.

So let us assume, finally, an Einsteinian universe for our twins. As before, when Terence actually sees the event of his sister's arrival at Alpha Centauri, his own clock registers 16 years. But, because of time dilation, Astrid's clock registers only 8 years, and thus *appears* to Terence to be running slow by a factor of 2, i.e. 1/2 as fast as his. On the return leg, Terence sees his sister's clock advance 8 years whilst his only advances 4; so the clock (and the aging processes of his sister and everything moving with her) *appear* to be running fast by a factor of 2. Despite these appearances, of course, Terence (who is now a whiz at physics) can *infer* that in each case the effect is 0.8 times what would be expected from the Doppler effect alone: a lag by a factor of 5/8, multiplied by 0.8, gives 1/2; a speeding up by a factor of 2.5 times 0.8, yields 2. Thus he *infers* that Astrid's clock is running slow because of time dilation.

Astrid, on the other hand, on looking back to Earth as she is arriving at Alpha Centauri 8 years later, sees the Tellurian clock register 4 years, as before. By Astrid's reckoning, the image has travelled eight years to get to Alpha Centauri, so Terence's clock *appears* to Astrid to be running slow by a factor of 2. On the return leg, Astrid sees her brother's clock advance its remaining 16 years whilst hers only advances 8; so the clock (and the aging processes of her brother and everything moving with him) appear to be running fast by a factor of 2. Again, she can calculate that since the effects should have been 5/8 and 2.5 if they were due to the Doppler effect alone, the difference is due to the fact that Terence's time is slowed relative to hers by the time dilation factor 0.8. (During her 8 years on the outward leg, she sees Terence's clock move 4 years when it should have moved 5 by the Doppler effect alone, since 8 times the Doppler effect of 5/8 = 5; on the way back she sees Terence's clock move 16 years instead of the 20 that would be 8 times the Doppler effect of 2.5). Thus the situation is entirely symmetrical: while they are in relative motion, each twin suffers an *inferred* time dilation, a slowing-down of the aging process, from the point of view of the other. And in terms of what *appears*, they both see their twin sibling's clock running slow by a factor of 2 while they are moving apart, and running fast by a factor of 2 when they are approaching one another.

This shows us that the scenario depicted is entirely consistent. But how does it resolve the paradox? If everything is symmetrical, then why don't the twins age by the same amount? The usual (and correct) explanation is as follows: although while the twins are in constant relative motion the situations are indeed perfectly symmetrical, this is not so for their journeys or paths through spacetime as a whole. For in this thought experiment, the terrestrial twin Terence undergoes no acceleration, whilst his adventurous astronaut sister must decelerate through –0.6c on arrival at Alpha Centauri, and then accelerate through another –0.6c to the same speed in the opposite direction. On the other hand, in the idealized conditions of the thought experiment, any time dilations due to the accelerations are ignored. But although this explanation is perfectly correct, it leaves a lingering sense of puzzlement. If the difference in the ages of the twins is not due to any time dilation caused by acceleration, and yet while they are in inertial motion relative to one another each sees the other's time dilated by the same factor, *how does the asymmetry in the paths taken result in a difference of time elapsed for each twin? How is a difference in paths even relevant to the situation*? To many people, this has suggested that the difference in the twins' ages *is* due to a time dilation caused by acceleration.

4. Modified Twin Paradox

Thus it is often said that the reason for the discrepancy in the twins' ages is that whereas Terence is in inertial motion throughout, Astrid is the one who really moves because of her acceleration, although this acceleration lies outside the scope of the theory. This seems to imply that it is Astrid's (here instantaneous) non-inertial motion that is responsible for the dilation.[10]

[10] See for instance J. J. C. Smart, (1968, 231): "The clock paradox comes from the following fallacious bit of reasoning. In our calculations we have taken Jack to be at rest and Jim to be moving with a velocity of

Some even go so far as to claim or imply that Special Relativity applies only to systems in inertial motion[11], and that a proper resolution of the paradox must therefore involve General Relativity.[12] But this is incorrect. On the one hand, Special Relativity is perfectly applicable to accelerated motions, and on the other, although the fact of Astrid's acceleration is a necessary condition for their taking different paths through spacetime, the time dilation is due to the different paths, not an effect of the acceleration itself.[13]

To see this, we can construct a journey for each of the twins with equivalent non-inertial paths as follows. Suppose the trusty Terence tires of his terrestrial tenure, and takes residence in Telstar, a nearby space-station, a doughnut shaped ship that simulates gravity by rotating. To relieve tedium, Terence sets it spinning very fast about an inertial point, so that for precisely the period (say, 2 years, in the terrestrial frame) in which Astrid is decelerating at –0.6c a year, and undergoing a corresponding time dilation due to this acceleration (leaving her D years younger, where $0<D<2$), Terence undergoes an exactly corresponding time dilation due to his rotational acceleration. Now when Astrid returns, she will be between 16 years and 18 years older, and Terence will be between 20 and 22 years older, and they will differ in age by exactly 4 years. In

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either +*v* or –*v* relative to him. Equally, it is said, we could take Jim to be at rest… The fallacy in the reasoning is that the first calculation (showing Jim to be younger than Jack) was correct, because Jack has been in the same inertial system throughout. However Jim had to be accelerated at Alpha Centauri…"

[11] To cite two contemporary examples from the world wide web:. "However, this resulted in a limitation inherent in Special Relativity that it could only apply when reference frames were *inertial* in nature..." (http://en.wikipedia.org/wiki/Inertia); "This dilemma highlights a limitation of the Special Theory of Relativity that we have already alluded to. It only applies to observers in uniform motion, and not to accelerated frames." (http://theory.uwinnipeg.ca/mod_tech/node141.html).

[12] Cf. this analysis on the *Encyclopedia Britannica* internet site: "The answer is that the paradox is only apparent, for the situation is not appropriately treated by special relativity. To return to Earth, the spacecraft must change direction, which violates the condition of steady straight-line motion central to special relativity. A full treatment requires general relativity, which shows that there would be an asymmetrical change in time between the two sisters. Thus, the "paradox" does not cast doubt on how special relativity describes time, which has been confirmed by numerous experiments." http://qa.britannica.com/eb/article-252886.

[13] One suspects that Einstein himself unwittingly contributed to this misunderstanding by using arguments in his (1918) from General Relativity to defend the consistency of the Special Theory. But in fact what Einstein is defending is not the self-consistency of SR attacked by Dingler, but the consistency of the account of *time dilation due to accelerated motion* in SR with the General Relativistic equivalence of acceleration with gravity.

the reference frame of the inertial point near earth, exactly 22 years will have passed.[14] But in their paths through spacetime, the twins will have been in inertial motion for the same 20 years with respect to that point and its inertial frame, and will have undergone time dilation due to their accelerations for the same two years in that frame. Yet one is still 4 years older than the other. It follows that it can't be said that Special Relativity applies only to systems in inertial motion —if that were so, there could be no explanation of the Twin Paradox in the theory. But we have just so explained it! Thus *the difference between the ages of the twins is not due to one's being in inertial motion, the other not*. Both their ages are true measures of time, in the original thought experiment, as well as in this modified version.

The correct conclusion is that it is not any difference between inertial frame-times that accounts for the difference in the twins' ages, but the difference in their paths through spacetime. It is the time elapsing along a particular path in spacetime that measures how fast the processes traversing that path are going, how fast the people or things undergoing them are aging, how fast they are becoming. In the non-Euclidean metric of Minkowski spacetime, it is the longest, not the shortest, time interval between two spacetime points that is given by the straight line in spacetime connecting them.[15] The longer the spacetime path between them, the shorter the time elapsed along that path. In the original thought experiment, Astrid travels along two sides of a triangle, and Terence by the remaining side; in the modified version, Astrid's straight lines are joined by a curve, while Terence's straight line is interrupted by a spiral of the same length. In each case Astrid's path is longer, and the time elapsed shorter. It follows that it can't be said, as one often reads, that the duration of processes in relativity theory is relative to an inertial frame. In the sense of time lapse that is relevant to the twin paradox —how much time

[14] Of course, the physical situation could be made even more realistic, if desired, by having Astrid accelerate away from Earth to his speed of 0.6c, and then decelerate to zero on return. But again, this could be compensated for by having Terence spin his Telstar for the same period.

[15] This is not to say that this feature of the Minkowski metric is *causally responsible* for the fact that the time lapse is longest along an inertial path, an interpretation against which Harvey Brown has argued in detail in his (2005).

elapses for each twin— *it is simply false that time lapse is frame-dependent,* i.e. depends on the inertial frame adopted. Indeed, the duration of each twin's journey through spacetime is an invariant measure: it is the same in all inertial reference frames.

5. PROPER TIME AND PROPER LENGTH

As I stated at the start of this essay, there are in fact two different measures of time in relativity theory: they have different formal measures, and different ontological baggage. This parallels the case for mass. In each case, what in classical physics had been thought to be a univocal or absolute property of the system turned out to be degenerate. For in the transition to Einstein-Minkowski physics mass bifurcates into the relativistically invariant *proper mass* m_0, and the *relative mass* μ, or mass in an inertial frame in which it is moving at a speed $v = \beta c$, whose quantity is a factor $\gamma = (1 - \beta^2)^{-1/2}$ times the proper mass.

But, as I have suggested above, I believe much of the confusion about relativity comes from interpreting proper time as if it is simply the relative time of an observer in her own rest frame. This misinterpretation is encouraged by the analogy with mass, but even more so, I will now suggest, by reading the case of time or *duration* as an exact analogue of that of space or *length*.[16] For the degeneracy of time in relativity theory is paralleled by a similar bifurcation in the concept of length. A body that is moving at a speed $v = \beta c$ with respect to a given inertial reference frame will, as already discussed, undergo a length contraction in the direction of its motion, so that its length $L = L_0 / \gamma$, where L_0 is its *proper length*. The latter is defined as its length in the rest frame: if $v = 0$, $L = L_0$. Analogously, it may be thought, any periodic processes associated with the body will suffer a time dilation, so that $t = \gamma t_0$, with the result that in the rest

[16] I am indebted to Storrs McCall (private communication) for suggesting to me the relevance here of the analogy with proper length. I am also indebted to Kent Peacock for helping me eradicate some infelicities in my discussion of this in an earlier draft.

frame where $v = 0$, $t = t_0$. *Proper time*, then, it may be asserted, *is just t_0, the time coordinate as measured in the rest frame*.

But this it is not! Proper time was introduced by Hermann Minkowski in his famous 1908 paper (Lorentz et al. 1923, 73-91) as follows. If at any point P (*x, y, z, t*) in spacetime we imagine a worldline running through that point, the magnitude corresponding to the timelike vector *dx, dy, dx, dt* laid off along the line is

$$d\tau = \sqrt{(c^2dt^2 - dx^2 - dy^2 - dz^2)}/c$$

Proper time is now defined as the integral of this quantity along the world line in question. Introducing the concept, Minkowski wrote: “The integral $\tau = \int d\tau$ of this quantity, taken along the worldline from any fixed starting point P_0 to the variable endpoint P, we call the proper time of the substantial point at P.” (85) As he proceeded to explain, *x*, *y*, *z* and *t* —the components of the vector OP, where O is the origin— are considered as functions of the proper time τ, and the first derivative of the components of this vector with respect to the proper time, $dx/d\tau$, $dy/d\tau$, $dz/d\tau$ and $dt/d\tau$, are those of the *velocity vector* at P.

It is a consequence of this definition that the element of proper time $d\tau$ is not a complete differential. Arnold Sommerfeld, in his notes appended to Minkowski’s 1908 paper when it was reprinted in a book (Lorentz et al. 1923, 92-96), remarked that Minkowski had mentioned this to him. He comments:

> [T]he element of proper time $d\tau$ is not a complete differential. Thus if we connect two world-points O and P by two different world-lines 1 and 2, then
>
> $$\int_1 d\tau \neq \int_2 d\tau$$

> If 1 runs parallel to the t-axis, so that the first transition in the chosen system of reference signifies rest, it is evident that
>
> $\int_1 d\tau = t, \quad \int_2 d\tau < t$
>
> On this depends the retardation of the moving clock compared with the clock at rest. (94)

Evidently, Sommerfeld had already resolved the twin (clock) paradox in 1923 in essentially the same terms as I have given above.

What is crucial to this resolution is that the proper time calculated along a given path in spacetime is an *invariant quantity*: it retains the same value under transformation of inertial frame. It is for this reason that it "can claim the prerogative of representing the objective lapse of time", to use Gödel's own words (558), and thus undermines his argument from the relativity of simultaneity to the unreality of time. Of course, Gödel assumed that an objective lapse would have to consist in a *global* plane of becoming, and therefore could not be relative to spacetime path; but, according to the point of view I am advocating here, this assumption is unwarranted in relativistic physics, where becoming is *local*, and dynamical change is parametrized by proper time, not co-ordinate time. It remains the case, of course, that the proper time is a maximum in the rest frame of an inertially moving object, and that in this circumstance it is numerically equal to the co-ordinate time . For when $v = 0$, $\beta = 0$, so that $\gamma = (1 - \beta^2)^{-1/2} = 1$, and $\tau = t_0$. But this is only numerical equality, not identity. It corresponds to the fact noted above that the longest time interval between two spacetime points in timelike separation is given by the straight line in spacetime connecting them. All other paths, whether the two inertial paths of the original Twin Paradox thought experiment, the paths of the Modified Twin Paradox incorporating a curve of deceleration and a spiral, or even a steady curve representing the travelling twin gradually slowing up turning round and returning, are shorter. But by the same token, Special Relativity is perfectly able to account for these non-inertial paths, and for each of them the proper time would be calculated by an integration along the path in question, not by the difference in time co-ordinates in any inertial frame. If proper time were the time co-ordinate in an inertial frame at rest, t_0, it would not be applicable to such curved paths. In contrast, proper length is the length of an object —a metre stick, say— in a specific frame of reference, namely, the inertial frame in which it is at rest.

Still, it may be objected, proper length is nevertheless also an invariant quantity. Just as the length of a path joining two events in timelike separation is invariant under change of frame, so is the length of a curve joining two events in spacelike separation. Indeed, it is often argued that the analogy between it and proper time is perfect: “proper length is the invariant interval of a spacelike path whereas proper time is the invariant interval of a timelike path”.[17] Thus, it is suggested, the definition of proper length should be generalized so that it is the exact analogue of proper time: a line integral along a curve joining two spacelike separated events. But a little further reflection shows that this cannot be right: an arbitrary curve joining two spacelike separated events is not generally a length. It is only a length if all the points on the curve are simultaneous in some given reference frame. And while the path integral along such a curve is indeed independent of the choice of reference frame, it has no particular physical significance. It does not even represent a *path*, in the normal acceptation of a path as a series of positions that can be successively traversed —as, for instance, by Harvey Brown’s waywiser (Brown 2005, front cover, p. 8)— for such an interval is timelike. Proper length is correctly defined as the path integral, not along *an arbitrary curve* joining the endpoints of the metre stick at the same time, but along *the shortest curve*, which is a straight line joining them in the frame at which they are at rest. If (elapsed) proper time were the strict analogue of this, it would be the longest time between two timelike separated events, which would be the time in a frame of reference at rest,

[17] Quoted from an article on proper length in Wikipedia (http://en.wikipedia.org/wiki/Proper_length: May 5, 2007). The author suggests a generalization of proper length so that it is given by the line integral $L = c \int_P \sqrt{[-g_{\mu\nu}\, dx^\mu\, dx^\nu]}$, where $g_{\mu\nu}$ is the metric tensor for the spacetime with +--- signature, normalized to return a time.

i.e. the co-ordinate time. It is precisely this interpretation that I am attacking. Because proper length is the interval between two events *at the same co-ordinate time*, it is specific to a particular reference frame.

Thus *proper time* has a fundamentally different character from proper length. Although both are invariant under change of frame, proper length is the length of an object in its own rest frame, whereas proper time is independent of frame. In this respect proper length is analogous to proper mass. (It differs from the latter, however, in that proper mass seems to be an essential characteristic of an elementary body (such as an electron), whereas proper length is a contingent one.) At any rate, there is a fundamental dissymmetry between duration and length in Special Relativity, somewhat obscured by talk of their embodiments in observers' clocks and rods. For whereas an observer's clock measures proper time elapsed along a path, a dynamical variable specifiable independently of reference frame, the proper length of the observer's measuring rod is specific to the inertial frame in which the observer is at rest. Thus we see that, ironically, there is a sense in which Minkowski's introduction of proper time undermines his famous pronouncement at the beginning of his paper about the demise of time: "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality". (75)[18]

Interestingly, this very lack of a frame-independent invariant length in relativity theory leads to another paradox, although this paradox depends on a combination of quantum theory and relativity theory. Even though it is something of a digression from my main argument, it seems to further accentuate the disanalogy between space and time. The point is this: in the attempt to come up with a theory of quantum gravity, all researchers are agreed on the fundamentality of

[18] Minkowski's judgement is echoed by Einstein in his essay "The Problem of Space, Ether and the Field in Physics": "Hitherto it had been silently assumed that the four-dimensional continuum of events could be split up into time and space in an objective manner... With the discovery of the relativity of simultaneity, space and time were merged in a single continuum ... " (1954, 281-82).

the Planck length, $L_P = \sqrt{\{\hbar G/c^3\}}$, where $\hbar$ is Dirac's constant (Planck's constant divided by 2π), and G is the gravitational constant: this is extremely small, about 1.6×10^{-35} metres. In some sense this should represent the smallest length there is, and be so for all possible observers. And this is where the paradox gains purchase: according to Special Relativity, there is no frame-independent invariant length. If the Planck length L_P is regarded from a frame of reference moving with respect to it at $v = \beta c$ it will, as already discussed, be contracted in the direction of its motion, so that its length will be less, namely $L = L_P / \gamma$. But by hypothesis, nothing is shorter than the Planck length. In other words, Einstein's relativity theory —which in any case may be expected to break down at this scale— is in contradiction with the posited invariance of the Planck length, since in it there is no preferred reference frame. This paradox was proposed by Giovanni Amelino-Camelia in 1999, who at the same time proposed a solution to it with the so-called theory of Doubly Special Relativity (or DSR).[19] The principal idea of this theory is that there is not just one invariant constant, the speed of light c —which is now the speed of very low energy photons— but also the Planck length. If this theory is true, of course, the invariance of the Planck length is analogous to that of c, not to that of the proper time, since it is a universal constant, whereas proper time is a variable. But whatever the fate of DSR, it seems that time and space are on very different footings in relativity theory.

I am by no means the first to point out the radical implications of relativity theory for our understanding of time. As Milič Čapek has stressed in several publications (1966, 1975, 1976), the invariance of Minkowski's relations of being in the absolute past or future of an event means that in relativity theory the role of time is strengthened and made more distinct than in classical physics. The distinction between proper time and coordinate time is stressed by Larry Sklar in

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[19] Giovanni Amelino-Camelia (2001); see Smolin (2006, 365). H. S. Snyder had already resolved the incompatibility between the existence of a minimum length and the requirement of Lorentz invariance in his (1947), positing two invariant scales and a non-linear basis for the Poincaré algebra. See also Joy Christian (2006), whose theory posits only one observer-independent conversion factor, the inverse of the Planck time, with c emerging as an invariant but derivative factor.

his treatment of the clock paradox;[20] and Kent Peacock (1992), has also discussed the paradox in terms of a comparison between the proper times of the twins while they are spatially distant. But perhaps the clearest explanation of the distinction between "time co-ordinate" and "proper time" and its significance was given by Howard Stein:

> Proper time is not a quantity attached to space-time points or to pairs of space-time points; it is in this respect a notion utterly different from the quantity "time" or "time interval" of pre-relativistic theory... The fundamental physical role of proper time comes from the principle (here stated roughly) that whenever a process takes place along a well-defined line of space-time ("world-line"), the time rates in the dynamical principles that govern that process are to be understood in terms of the proper time along that line (and *not* in terms of a "time coordinate"...)[21]

Yet it seems to me that the significance of this degeneracy of time in relativity theory is still largely unrecognized. Philosophers and physicists continue to write as if it is the time co-ordinate function, or time in relation to an inertial observer, and not proper time, that measures the duration of processes in relativistic physics. This is implicit in all discussions that agree with Gödel in construing the objective lapse of time in terms of an infinity of layers of "now", with these planes of simultaneity picked out by the time co-ordinate function in an inertial reference frame, such as the arguments of Putnam, Rietdijk and Maxwell discussed above.

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[20] Sklar (1974, 268) correctly points out that, whereas "'co-ordinate time' between two events is relative to a given inertial frame", "[p]roper time is defined only for events at timelike separations and only relative to a particular spacetime curve between the events. On the other hand it is an invariant notion." Unfortunately, though, he goes on to claim that anyone who wishes to assert that future events are not real relative to the assertor is forced by the Putnam-Rietdijk argument to admit that such notions are "just as relative to an inertial state of motion of the assertor and just as 'nontransitive across observers in different states of motion' as we have made the simultaneity relation." (275).

[21] Stein (1968, 11, fn. 6). This quotation from Stein was my starting point in the line of argument for his paper. Cf. also p. 16: "... '*a time co-ordinate'* is not '*time*.' Neither *a* nor *b* is, in any physically significant sense, 'present' (or past) for any observer at *c*—regardless of his velocity—for neither *has already become* for *c* (nor has *c* for them); but *a* has already become for *b*, and can influence it." [Here *a* and *b* are connectible by a time-like vector *ab*, the other pairs by space-like vectors *ac* and *bc*.]

6. CONCLUSION: PROCESS AND BECOMING

Finally, I want to return to Gödel's attack on the reality of time based on his argument from General Relativity. It is not generally recognized that this same conception of time lapse in terms of the planes of simultaneity parametrized by a time coordinate function underlies this argument too.

Having stated his objections to the relativity of time lapse, Gödel considers a way of avoiding this relativity first proposed by the astronomer James Jeans in 1935. This is founded on the observation that the expansion of the universe does after all allow (in principle) the singling out of one set of preferred reference frames, relative to which a notion of cosmic time can be defined, namely the family of frames tracking the mean motion of matter. As Gödel reports, Jeans had proposed this as a way of recovering "the intuitive idea of an absolute time lapsing objectively".[22] At any rate, when we talk about the Big Bang having occurred 15 billion years ago, it is with respect to such a cosmic time function that the age of the universe is being calculated.

It is against this Jeansian scenario that Gödel directs his argument from General Relativity. If the incongruity of having time lapsing at different rates for different observers is to be circumvented by an appeal to a cosmic time function, what if there are scenarios where no such cosmic time function can be constructed? This motivates his construction of solutions to Einstein's field equations representing static, spatially homogeneous universes, rotating with respect to the totality of galactic systems. In these "rotating universe" solutions, such "an *absolute* time does not exist" (1949a, 447): that is, because of the existence of closed timelike

[22] Gödel (1949b, 559); James Jeans, (1935, 22-23). As Gödel observes, the "mean motion of matter" is not a very precise notion, and is contingent on facts about our cosmos: "What may be called the 'true mean motion" is obtained by taking regions so large that a further increase in their size does not any longer change essentially the value obtained. In our world this is the case for regions including many galactic systems" (1949b, 559, n. 7). This approximation could perhaps be improved, but would still involve "introducing more or less arbitrary elements (such as, e.g., the size of the regions or the weight function to be used in the computation of the mean motion of matter)" (560, n. 9).

curves ("time loops") through every spacetime point of the solution, "there can be no cosmic time coordinate *t* in $\mathcal{M}$ which increases along every future-directed time-like or null curve" (Hawking and Ellis 170). The *coup de grace* —"strengthening further the idealistic viewpoint"— is then provided by the fact that the existence of these time loops implies that in Gödel's rotating universes it would be possible (in principle) "by making a round trip on a rocket ship in a sufficiently wide curve" for someone "to travel into any region of the past present or future", and in particular "into the near past of those places where he has himself lived." (1949b, 560-561) This opens up the whole Pandora's box of paradoxes concerning killing one's own parents or grandparents and thus preventing one's own conception, towards which much philosophical attention has been directed.

Since Gödelian universes are causally pathological, physicists have preferred to exclude their possibility by stipulation. Thus Hawking and Ellis lay down as a postulate "that space-time satisfies what we shall call the *chronology condition*: namely, that there are no closed timelike curves" (1973, 189). An apparently stronger condition is the *causality condition*, which holds if there are no closed timelike or null geodesic curves, although Hawking and Ellis give an argument to prove that "in physically realistic solutions, the causality and chronology conditions are equivalent" (192). Stronger than this is the *strong causality principle*, which holds at some point *p* "if every neighbourhood of *p* contains a neighbourhood of *p* which no non-spacelike curve intersects more than once" (192). Any of these conditions, laid down as a condition for the physicality of a universe (and thus as a constraint on the viability of solutions to Einstein's field equations), will preclude Gödel's cylindrical universes by *fiat*. But it does not seem likely that Gödel would have found such attempts to parry his arguments any more compelling than the Jeansian strategy he was refuting. His intuition is essentially Kantian: if time lapse is objective, it must be a feature of any possible universe, including those that do not expand. The fact that ours is an expanding universe is a contingent fact —at least, according to our current understanding: there is no known way of deriving the fact of the expansion of the universe from physical law. Likewise, if there is no way of deriving the causality condition from first principles., then, as a rebuttal of Gödel, it has no more force than the brute empirical fact of the expansion of the universe. An *a posteriori* axiom does not have the requisite power to establish what must be true *a priori* of any possible universe. If it is possible in principle for there to be a universe to which time lapse is not applicable, then this is enough, by Gödel's lights, to refute the objectivity of time lapse.

But does Gödel's argument prove that time lapse is self-contradictory? The scenario he depicts is of two space travellers traversing different paths through his cylindrical spacetime — one from P to Q (perhaps an indefinitely small distance away) along a worldline, one from Q in a timelike curve back to P— in such a way that when they meet back up their clocks do not agree on how much time has elapsed. Gödel argues that if there are 3-spaces "which are everywhere spacelike and intersect each worldline of matter in one point", then "time measured along the world lines of matter in their positive direction would yield a coordinate system with the property that the 0^{th} coordinate always increases if one moves in a positive time-like direction" (1949a, 447, 449), in contradiction to the existence of the time loop scenario described, "which implies that all coordinates of the initial and the endpoint of a time-like line [e.g. here any of the timelike lines from P through Q back to P] are equal in certain cases" (449). That is, if time lapse is to be measured by a (any) time co-ordinate function, then, despite the time taken for the traveller's trip, it will come out to be 0 according to that time-coordinate function.

But if this proves time lapse unreal, then this is already proven in Special Relativity, by the case of the Twin Paradox considered above! The problem with Gödel's formulation, as I hope should be clear by this juncture, is that it fails to appreciate the degeneracy of time: time lapse is not represented in relativity theory by the time co-ordinate function, but by proper time.[23] This is the time as measured for the paths through spacetime, and, as we have seen, these will not in general be the same in any spacetime where the motions along those paths involve asymmetric accelerations. The time elapsed for the traveller traversing a time loop, as measured by the proper time, will generally be quite considerable, whereas for a traveller who has travelled from P to a point Q an arbitrarily small spacetime interval away, the proper time elapsed will be arbitrarily close to zero. Thus, as paradoxical as the scenario depicted by Gödel is, it does not refute the possibility of time lapse. It precludes time lapse as he conceived it, in terms of global planes of simultaneous becoming; but not time lapse conceived, as I have urged is implicit in relativity theory, as the unfolding of processes along worldlines. What, then, are the implications of Gödel's scenario for our understanding of time?

I think it is crucial to remember here that spacetime in general relativity theory is not a background, nor is it a perduring space which we may imagine a traveller travelling through and exploring. One cannot *have* the spacetime and *then* superpose trajectories or events onto it: this is to confuse spacetime with a perduring background space. Any process or events must be represented in the spacetime. So in this case it is legitimate to ask: does a worldline looping from P through Q and back to P represent a possible process? Notice that this is to pose a question that is slightly different from the usual one about the possibility of confronting one's younger self: I am asking, is it possible for a continuous worldline to loop from P back to the same event P? Could one have a space traveller, or any other object large enough to bear traces of aging, both bear and not bear those traces at P? Clearly not.[24]

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[23] The same point has been stressed by Dennis Dieks in his (2006): "The rate of these local processes is determined by the amount of *proper time* between events, and not by differences in cosmic time" (167).

[24] Commenting on such scenarios as travelling such a loop and preventing oneself from setting out in the first place, Hawking and Ellis comment: "Of course there is a contradiction only if one assumes a simple notion of free will" (1973, 189). That this is incorrect is shown by my example of an object traversing a time loop through some point P, which would have to both bear and not bear traces of aging at P.

Now, since aging entails an irreversible process, this might seem to suggest that perhaps a simple reversible process *could* lie on such a loop. I would argue, however, that not even this is possible: an individual process (and here we are idealizing a process as 1-dimensional) must be from some one point-event *a* to another *b*, and cannot also be from *b* to *a*, without violating the idea of what a process is. That is, the relation of becoming —one individual event x's coming out of another y, xBy— is intrinsically asymmetric: $(\forall x, y)\ (xBy \rightarrow \neg yBx)$.[25] Now, of course, transitivity might fail: one might have local becoming along "sufficiently small" segments of a worldline, a piece-wise, fragmentary process of local becoming along individual worldlines, not adding up to "global becoming along the loop".[26] But such a non-additivity of processes seems (to me at least) to be as contrary to our intuition of process as a failure of asymmetry would be. If this is so, then in the spacetimes that are solutions to Einstein's field equations, this requirement of the asymmetry and transitivity of process on lines that represent processes entails the chronology condition of Hawking and Ellis: there must be no closed timelike curves.

Now if this analysis is correct, I believe we have a full answer to Gödel. He is right to demand that if time lapse is to be counted as objective, it should both be invariant and be a feature of any possible universe. We have seen that the objectivity of time lapse is guaranteed by its being measured by the proper time, with proper time invariant under change of inertial frame. But in addition, in order for a timelike curve to represent the trajectory of a process, it cannot, because of the asymmetry and transitivity of becoming, be closed. But this means we have an argument for the chronology condition: it is an *a priori* condition for representing a possible process in spacetime.

[25] This is a different question than that of the reversibility of processes, which concerns whether *types* of processes of a given kind always occur —whether nomologically or *de facto*— one way round with respect to an already given time-direction.

[26] This was suggested by the anonymous referee of this paper.

This perspective seems to have implications for modern attempts to eliminate time from physics. Time, it entails, is not just a co-ordinate, appearing in the metric on a par with the three dimensions of space except for the factor of *i*, (Stephen Hawking's "imaginary time": see his 1988, 134-139). It is fundamentally dynamical, tracking the evolution of all processes in the universe, and so cannot be rolled over into a space coordinate as we trace it backwards towards the universe's origins. The causal/chronological structure of timelike lines and worldlines are not symmetrical with spacelike lines in such a way as to be transformable away. In several approaches to quantum gravity, as Lee Smolin has argued, "*causality itself is fundamental* —and is thus meaningful even at a level where the notion of space has disappeared" (Smolin 2006, 241) To this we may add: if it is indeed the case that time lapse is a necessary condition for process, then a world without time is a world without process. Now one is not obliged to depict interactions in particle physics, say, using the space-time representation in quantum physics: sometimes it is more convenient to use the energy-momentum representation. But to conclude from this that there is no time is to say that there is no process, and one wonders then what it is that is being represented. Similar misgivings, it would seem, should apply to Julian Barbour's claim there is no time in the most fundamental description of reality, the timeless universe he has dubbed "Platonia" (1999).[27]

To conclude: I have argued that time is degenerate in relativity theory. Co-ordinate time is used to track the synchrony of distant events, but it no longer has the classical role of tracking a worldwide hyperplane of becoming, as it did in classical theory. Instead it is proper time that measures time elapsed, and thus gives the true measure of the duration and rate of processes in spacetime. This bifurcation of time's roles is masked by a tendency to assimilate proper time to time in an observer's rest frame, by analogy with proper length, a tendency which is

[27] Lee Smolin (2000) has given a detailed analysis of Barbour's argument, arguing that its conclusion can be resisted only if one restricts quantum cosmology "to theories in which all observables are accessible to real observers inside the universe" (23), and investigating in detail what this entails for cosmology.

encouraged by unwarranted talk of an observer "inhabiting an inertial frame" and "experiencing" the events which are simultaneous with his or her state of consciousness. But whereas proper length is specific to a rest frame, proper time is not; its intervals are path-dependent, frame-independent, and invariant under change of reference frame. It is this proper time that measures the time elapsed for travellers in spacetime, which consideration is sufficient to resolve the Twin Paradox, as is shown with particular attention to what the twins could actually observe and infer about each other's times. It also disposes of Gödel's arguments for the ideality of time from the existence of closed timelike curves in certain General Relativistic spacetimes. On the contrary, it is argued, the condition that every timelike curve represent a possible process yields a justification for the chronology condition: because of the intrinsic asymmetry and transitivity of becoming, a closed time like curve cannot represent the path of a possible process.

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References

Amelino-Camelia, Giovanni (2001), "Testable Scenario for Relativity with Minimum Length", *Phys Lett.* B510 [arXiv:hep-th/0012238].

Arthur, Richard T. W. (1982), "Exacting a Philosophy of Becoming from Modern Physics," *Pacific Philosophical Quarterly* 63, 2, pp. 101-110, April.

Arthur, Richard T. W. (2006), "Minkowski Spacetime and the Dimensions of the Present," chapter 7, pp. 129-155 in Dennis Dieks, ed. (2006).

Barbour, Julian (1999), *The End of Time*. (Oxford/New York: Oxford University Press).

Brown, Harvey (2005), *Physical Relativity: Spacetime structure from a dynamical perspective* (Oxford: Oxford University Press)

Čapek, Milič (1966), "Time in Relativity Theory: Arguments for a Philosophy of Becoming," pp. 434-454 in J. T. Fraser, ed., *Voices of Time*, (New York: Brazilier).

Čapek, Milič (1975), "Relativity and the Status of Becoming," *Foundations of Physics* **5**, 4, 607-617.

Čapek, Milič, ed. (1976), *Concepts of Space and Time*. (Boston: D. Reidel)

Clifton, Rob, and Hogarth, Mark (1995), "The Definability of Objective Becoming in Minkowski Spacetime", *Synthese* **103**: 355–387.

Davies, P. C. W. (1989), *The Physics of Time Asymmetry*. (University of California Press).

Davies, Paul (1995), *About Time*. (New York: Touchstone).

Dieks, Dennis (2006), "Becoming, Relativity and Locality," pp. 157-176 in Dieks (ed.) 2006.

Dieks, Dennis , ed. (2006), *The Ontology of Spacetime*. (Amsterdam: Elsevier).

Dingle, Herbert (1972), *Science at the Crossroads* (London: Martin, Brian and O'Keefe).

Dorato, Mauro (2006), "The Irrelevance of the Presntist/Eternalist Debate for the Ontology of Minkowski Spacetime," pp. 93-109 in Dieks (ed.) 2006.

Einstein, Albert (1918), "Dialog über Einwände gegen die Relativitätstheorie", *Die Naturwissen–schaften* **48**, pp. 697-702, 29 November; an English translation exists on the web at http://en.wikisource.org/wiki/Dialog_about_objections_against_the_theory_of_relativity.

Einstein, Albert (1954), *Ideas and Opinions* (New York: Crown Publishers).

Gödel, Kurt (1949a), "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation," *Reviews of Modern Physics 21*, 3, July 1949, 447-450.

Gödel, Kurt (1949b), "A Remark about the Relationship between Relativity Theory and Idealistic Philosophy", 557-562 in *Albert Einstein: Philosopher-Scientist*, ed. Paul Arthur Schilpp. New York: Tudor.

Grünbaum, Adolf (1976), "The Exclusion of Becoming from the Physical World", pp. 471-499 in Čapek, Milič (1976).

Hawking, Stephen W. (1988), *A Brief History of Time*. London: Bantam Press.

Hawking, S. W. and Ellis, G. F. R. (1973), *The large scale structure of space-time*. Cambridge: Cambridge University Press.

Jeans, James (1935), *Man and the Universe*, Sir Halley Stewart Lectures.

Lorentz, H. A., Einstein, A., Minkowski, H. and Weyl, H. (1923), *The Principle of Relativity* (Methuen 1923; reprinted, Dover 1952).

Maxwell, Nicholas (1985), "Are Probabilism and Special Relativity Incompatible?" *Philosophy of Science* **52**: 23–43.

Peacock, Kent (1992), "A New Look at Simultaneity", in *Philosophy of Science Association 1992,* Volume I, D. Hull, M. Forbes, and K. Okruhlik (eds.). East Lansing: Philosophy of Science Association, pp. 542–552.

Putnam, Hilary (1967), “Time and Physical Geometry”, *Journal of Philosophy*, LXIV,, 8, April 27, 240-247.

Rietdijk, C. W. (1966). “A Rigorous Proof of Determinism Derived from the Special Theory of Relativity,” *Philosophy of Science*, XXXIII, 4 (December), 341-344.

Savitt, Steven (2006), “Presentism and Eternalism in Perspective,” pp. 111-127 in Dieks (ed.) 2006.

Sklar, Lawrence (1974), *Space, Time, and Spacetime* (Berkeley: University of California Press).

Smart, J. J. C. (1968), *Between science and philosophy*. New York: Random House.

Smart, J. J. C. (1980), “Time and Becoming” in *Time and Cause*, ed. Peter van Inwagen (Dordrecht: Reidel).

Smolin, Lee (2000), “The present moment in quantum cosmology: Challenges to the argument for the elimination of time”, preprint, pp. 1-29.

Smolin, Lee (2006), *The Trouble with Physics* (Boston/New York: Houghton-Mifflin.)

Snyder, Hartland S. (1947), “Quantized Space-Time”, *Physical Review*, **71**, 38-41.

Stein, Howard (1968), “On Einstein-Minkowski Space-Time”, *Journal of Philosophy* **65**: 5–23.

Yourgrau, Palle (1991). *The Disappearance of Time: Kurt Gödel and the Idealistic Tradition in Philosophy* (Cambridge: Cambridge University Press).