Time, Inertia and the Relativity Principle

Richard T. W. Arthur
McMaster University
Hamilton, Ontario, Canada

Abstract: In this paper I try to sort out a tangle of issues regarding time, inertia, proper time and the so-called "clock hypothesis" raised by Harvey Brown's discussion of them in his recent book, *Physical Relativity*. I attempt to clarify the connection between time and inertia, as well as the deficiencies in Newton's "derivation" of Corollary 5, by giving a group theoretic treatment deriving from work of J.-P. Provost. This shows how both the Galilei and Lorentz transformations may be derived from the relativity principle on the basis of certain elementary assumptions regarding time. I then reflect on the implications of this derivation for understanding proper time and the clock hypothesis.

Harvey's 'waywiser'

Harrison's H-1

I. Time and Inertia in Newton

In this paper I wish to pursue some reflections about time and inertia that I had begun in January 1990. At the time I was engaged in some intensive research on Newton's philosophy of time, and in the light of that research I was moved to reconsider an interesting little paper I had read by Jean-Pierre Provost on a group theoretic approach to time.[1] Although the group theoretic approach makes no appeal to the rods and clocks that feature in Harvey Brown's operationalist approach —and indeed HB does not seem to regard the group theoretic approach as very instructive pedagogically— some of the tentative conclusions I reached were nonetheless quite similar to his. This prompted me to take up those reflections again, in the hope that the contrast between the approaches might prove instructive.

The first point of contact between my ruminations of the early 1990's and HB's analysis is the recognition that in Newton's physics, *inertial motion grounds absolute time*. For where Barrow had been content to allow a clock to be "taken" as equable if it appeared to be so —for an instance, an hourglass, or the period of one of Jupiter's moons— Newton insisted that no coherent system of the world could be constructed unless an absolute true and mathematical time were presupposed, and according to which the astronomical equation of time would be calculated.[2] Thus in his scheme a body undergoing inertial motion traces out equal displacements in equal times, so that the spaces covered are true temporal measures: an inertial body is a clock beating absolute time. Equable time is thus internal to inertially moving bodies, that is for Newton, bodies moving in absolute space. As Julian Barbour has explained,[3] and as noted by HB[4], this led, through the work of Lange (1886) and others in the late nineteenth century, to the definition of "ephemeris time". There are actually two separate problems here concerning equable time, and it is worth remarking on this now

[1] Provost (1980). I had first studied Provost's paper in 1982-3. My studies on Newton in the early 1990s issued in the publications Arthur (1994) and (1995).

[2] See my (1995) for a discussion of these points; see also Barbour's (1989).

[3] Barbour (1989), p. 633.

[4] "Newton already saw the fact that absolute time cannot be *defined* in terms of the sidereal day. He anticipated the notion of 'ephemeris time which would be employed by the astronomers prior to the advent of atomic clocks" (Brown 2005, p. 19).

for its relevance to our later discussion of the clock hypothesis in relativity theory. The first is Newton’s problem, which is to determine equability by giving a dynamics in which forces can be identified, so that even if there is no body actually undergoing perfectly inertial motion and thus acting as an ideal clock, such an inertial motion can be calculated. Secondly there is Harrison’s problem, so eloquently described by Dava Sobel in her book *Longitude* (1995): to build an actual clock that so far as possible mimics an ideal, inertial clock. This, as the tragic story of John Harrison’s quest so eloquently attests, is no trivial task to accomplish.

But granting all this, there is a problem eloquently described by HB: what he calls the “miracle of inertia”. “Inertia, before Einstein's general theory of relativity,” writes HB, “was a miracle. By this I ... mean the postulate that force-free ... bodies conspire to move in straight lines at uniform speeds while being unable, by *fiat*, to communicate with each other.” (14-15) Otherwise stated, inertial motion is motion at a constant velocity in a straight line *relative to what*? This echoes Einstein’s question: “inertia resists acceleration, but acceleration relative to what?” (1954, p. 348). Within classical mechanics and the special theory of relativity alike, Einstein continues, “the only answer is: inertia resists acceleration *relative to space*. This is a property of space —space acts on objects, but objects do not act on space” (348). This way of regarding matters has led some commentators into treating the spacetime metric as an *entity*, the positing of which explains the miracle in question: force-free bodies follow the geodesics much as cartwheels followed the ruts in a Roman market road. According to HB, in 1924 Einstein himself still thought in this way:

> In 1924 Einstein thought that the inertial property of matter (to be precise, the fact that particles with non-zero mass satisfy Newton’s first law of motion, not that they possess such inertial mass) requires explanation in terms of the action of a real entity on the particles. It is the space-time connection that plays this role: the affine geodesics form ruts or grooves in space-time that guide the free particles along their way. (141)

Matters changed, however, with the realization that Einstein's field equations themselves provide the foundation for the geodesic principle, the principle that "the world-lines of force-free test particles are constrained to lie on geodesics of the connection" (141). For the fact that the covariant divergence of the stress energy tensor field $T_{\mu\nu}$ vanishes, "came to be recognized as the basis of a proof, or proofs, that the world-lines of suitably modelled force-free test particles are geodesics." (141) Thus in GR the geodesic principle is not a postulate, but a theorem; moreover, because external-force-free spinning bodies will deviate from geodesic paths, it "is not an essential property of localised bodies that they run along the ruts of space-time determined by the affine connection." *Pace* Einstein and others, the positing of spacetime as an entity does *not* serve as an explanation for inertial motion, whose explanation is in fact only forthcoming in GR.

Both classical mechanics and special relativity, on the other hand, take the existence of a class of inertial frames for granted. According to HB it is the very positing of the existence of these frames "relative to which the above conspiracy, involving rectilinear motions, unfolds" that constitutes the content of Newton's First Law. Lévy-Leblond said the same thing in 1976: "Indeed the very existence of such equivalent reference frames corresponds to the validity of the principle of inertia..." (1976), p. 271.

In what follows I am therefore going to take the existence of an equivalence class of inertial frames for granted, even though I grant that the justification of their existence is only forthcoming in GR. Moreover, following Provost, I will take the equivalence class of inertial frames as implicitly defining the *space* relative to which motions are determined. This is a *dynamical* definition, compatible with the approach of Lange, so eloquently described by Barbour in his magisterial book (1989). Indeed, Provost explicitly mentions the similarity of his approach to Lange's.[5] The idea is that bodies undergoing inertial motions trace out straight lines in space, vector displacements. But the fact that these lines are straight, while mathematically part of the understanding of the space as Euclidean, is only justified physically by reference to all other inertial systems.

---

[5] "A similar approach (apart from group ideas) due to Lange (1885) may be found in "Relativity and Cosmology" (Robertson and Noonan, 1968)." The relevant pages are pp. 69-79.

In regarding space simply as a group of translations, however, Provost is not very clear about how time is involved. As we shall see, it features in his account only in the axiom (Axiom 2 below) that requires translations achieved by an active boost of an inertial frame to follow the same additive law that translations achieved by a passive boost of a particle do within any one inertial frame. This is that if two such translations add to a third, then the combined times taken for the two translations separately is equal to the time taken for their vector sum. What is interesting about this is that Newton himself was very well aware of this equal time property as being inherent in displacements achieved by inertial motions. This is enshrined in his parallelogram law for the composition of motive forces: "A body acted on by [two] forces acting jointly describes the parallelogram in the same time in which it would describe the sides if the forces were acting separately".[6] Since this is based on his understanding of motive force impressed as being proportional to the change of motion effected in a given time, which is later enshrined in the *Principia* as Law II, Newton gives the parallelogram law there as a corollary of that Law. Interestingly, however, in one of his early manuscripts Newton had given a proof of the vectorial composition of motive forces (Arthur 2008b). This proof depends on velocities being derived by (what we would now call) an implicit differentiation of displacements, and gives a precise justification of the idea that two consecutive inertial motions will effect the same displacement as a third effected in the same time if the third is the diagonal of the parallelogram.

Here the time is assumed to be absolute time, although Newton grants that a relative time corresponding to an equable motion may stand in for it. The success of the entire dynamics which is built upon this presupposition of inertial motions beating out equable times is then the justification of that very presupposition: the definition of time is implicit in the first law. Newton assumes that there is a unique frame in which motions are absolute, even while granting that his dynamics will not be able to distinguish such a frame from one in uniform motion with respect to it. Henceforth I will talk about "the *stationary frame*", which for Newton means absolute space, but may be understood

---

[6] (Newton 1999, p. 417). A more literal translation would be: "A body [carried] by conjoined forces describes the diagonal of a parallelogram in the same time as [it would] the sides by the separate forces."

rather to mean any one of an equivalence class of inertial frames relative to which Newton's laws hold.

Accordingly, I begin with the following definition and axiom:

> *Definition 1*. *Inertial Frame*: an inertial frame (relative space) consists in a group of translations (inertial displacements). The displacements within each frame are effected by (point)-bodies undergoing inertial motions, tracing out straight lines in a Euclidean space.

> *Axiom 1*. *Principle of Inertia*: there exists an infinite class of equivalent reference frames in relative motion one to another, forming a (differentiable and connected) one-parameter group.

Any two inertial frames are obviously related to each other by their relative velocity, which is the single parameter in question. Now it is a well known theorem of group theory that for any (differentiable and connected) one-parameter group there is an additive parameter.[7] Even though the additive parameter here must be a function of the relative velocity of the frames, and can have the dimensions of a velocity, we cannot prejudge things and assume that it will be identical to the relative velocity —a fact whose significance will become clearer later. So as not to prejudge the issue, we will call the additive parameter characterizing the group of inertial frames *swiftness* and abbreviate it with the letter ζ:

> *Theorem 1*: the relative motions of the equivalent inertial frames (relative spaces) are parametrized by an additive parameter $\zeta(v)$. Their composition law is: $\zeta_{12} = \zeta_1 + \zeta_2$.

II. Time and the Relativity Principle

But the existence of a stationary frame —which is in fact any one of an equivalence class of inertial frames relative to which Newton's laws hold— together with the

[7] For a thorough discussion of this theorem and its pedagogical utility at a reasonably elementary level, see Lévy-Leblond and Provost (1979).

behaviour of a clock in such a frame, does not tell us anything about how time behaves in other frames from the point of view of the stationary frame. For this, we need to invoke a Principle of Relativity. Newton does invoke such a principle, his famous Corollary 5 of the laws of motion:

> The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.[8]

As Julian Barbour has observed, this was described as a hypothesis in earlier drafts, but (like the Parallelogram Law) is demoted to the status of a corollary in the *Principia* itself. Barbour thinks this is a mistake (p. 608). He quotes Newton's proof in full (from the Motte translation), noting that (as shown by the words he has italicised) the only interactions that Newton considers are those produced by collisions:

> For the differences of the motions tending towards the same parts, and the sums of those that tend toward contrary parts, are, at first (by supposition), in both cases the same; *and it is from those sums and differences that the collisions and impulses do arise with which the bodies impinge upon one another*. Wherefore (by Law II), the effects of those collisions will be equal in both cases; and therefore the mutual motions of the bodies among themselves in the one case will remain equal to the motions of the bodies among themselves in the other. (Barbour 1989, p. 577)

Barbour argues that this is a non sequitur: "there is ... no reason whatever why the strength of the interaction (the impulses) between two bodies (to consider the simplest case) should be the same when their centre of mass moves through absolute space with a uniform velocity as when it is at rest." (577-578) Newton's proof of Galilean invariance depends on his believing "the forces of interaction to be purely relative and

[8] Barbour quotes from the Motte translation, (Newton 1962, p. 20). In the new translation by Cohen and Whitman it reads: "When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion." (Newton 1999, p. 423)

directly derivable from the relative configuration of the matter in the world." (608). HB follows Barbour here, identifying Newton's "extra assumption" as follows:

> As measured by the observer at rest in the frame relative to which the laws of motion are initially postulated —let us call this the stationary frame— the forces will not depend on the collective state of uniform motion of the system of bodies under consideration. ... A similar assumption is being made about the inertial and hence gravitational masses." (37)

"Without this extra assumption," HB states, "it is not possible to derive the RP [i.e. Relativity Principle] from Newton's laws and Galilean kinematics." (38) Yet HB also claims that "it is clear that [Newton] was assuming two things. The first was the Galilean transformations between inertial frames in relative motion ... But significantly, Newton also presupposed the velocity independence of forces and masses." (37) This is confusing, for if the RP —which Barbour has identified with Galilean invariance— is *derivable* from Newton's laws together with the extra assumptions, it is hard to see why Newton would need also to *assume* it. A partial clarification is achieved, I think, if we distinguish the RP or Galilean invariance, on the one hand, from the Galilean transformations, on the other. But this still does not explain why HB takes Newton to be assuming the Galilean transformations as well as the extra assumption(s). At any rate, HB has his mythical "Keinstein" *postulate* the RP as well as Newton's extra assumptions in order to derive the Galilean transformations (38-40). Here the RP is interpreted as yielding the same accelerations under boosts of the system of bodies under consideration, given the same initial conditions (38).

Let me try to bring some order into all this. The essential point that Barbour is making is that Newton cannot have derived Galilean invariance of interactions from his laws alone, since Lorentz-invariant interactions are also compatible with them, and accelerations are not Lorentz-invariant. (I independently reached the same conclusion about Corollary 1 not following from Newton's Laws in my reflections in 1990.) But we have to be clear about what we mean by "the Relativity Principle". It cannot simply mean that the physics will look the same from within any one inertial frame, no matter what its state of motion, since the state of motion of a reference frame (relative space) is for Newton defined with respect to absolute space. Newton's Corollary V implicitly assumes a stationary frame with respect to which the relative spaces may be regarded as moving or at rest. But the vector addition of velocities and motive forces licensed by the parallelogram law applies only within that stationary frame and within any other equivalent inertial frame: this much Newton says and is entitled to say. Where he errs is in supposing not that the laws will look the same from within each relative space (inertial frame), but that the vector addition of velocities and motive forces in the moving frame, viewed from the stationary frame, will take the same form. Recall that within the stationary frame, the displacements in a given time will be as the velocities of inertially moving bodies. It is natural to assume that such displacements can instead be effected by bodies at rest in a moving relative space, but nothing guarantees that displacements produced by the motion of a stationary body in a space moving with velocity v will take the same time as a body moving in the stationary space with that velocity: time, remember, is tied to inertial motion of bodies within the stationary frame (or of bodies within another inertial frame); we do not yet have any criterion to dictate how it applies across inertial frames. Otherwise stated: the displacement produced in the stationary frame by a body at rest in a moving frame (relative space) will depend on the velocity of the relative space, but it will not necessarily be identical to the displacement produced by the same body moving in the stationary frame with that velocity. Granted, it is only in hindsight that we can see that this identification is not necessary: as we noted above, the fact that inertial transformations form a differentiable, connected, one-parameter group allows us to infer the existence of an additive parameter that is a function of the velocity, but not necessarily to identify it with the velocity.

So we need further assumptions in addition to our Principle of Inertia. First, following Provost, I will give the Relativity Principle the following concrete formulation: we assume that any displacement x within an inertial frame may be realized by the motion of a body at rest in a second inertial frame in relative motion to the first.

Axiom 2. Relativity Principle: any displacement x in any one inertial frame can be realized by an active boost of a point-particle at rest in a second inertial frame in motion relative to the first with a swiftness ζ.

Provost calls these "dynamical translations", as opposed to merely "geometrical" ones (456). I shall call them "boost displacements". In order for this to be physically realistic, we need to make the further assumption that the relative swiftness ζ of 2 inertial frames changes sign under change of sign of all the spatial axes:

Axiom 3. Space reflection property: if $x \to -x$, $\zeta \to -\zeta$. Or, $\zeta(-x) = -\zeta(x)$

(For simplicity's sake, I will not use full vector notation, but follow Provost in running the argument with one spatial dimension. It can readily be generalized to three.)

Now we need to consider time. As we saw, within an inertial frame Newton's parallelogram law demands that any two inertial displacements adding to a third must be effected in equal times. Therefore, according to the RP, so must any two boost displacements adding to a third, if they are successfully to realize the inertial ones. This gives us the following principle, original with Provost:

Axiom 4. Equal Time Principle: if three boost displacements x_1, x_2, x_3 satisfy $x_1 + x_2 = x_3$, then the displacements on both sides of the equation have been realized in equal times.

Finally, in order for these assumptions to succeed in implicitly defining time we also need to stipulate that at least some of the intervals so defined keep the same sign in all reference frames, otherwise causal processes will be impossible:

Axiom 5. Causality Condition: there exist time intervals that are invariant under transformations associated with any swiftness ζ.

Now the intriguing thing about Provost's approach is that on the basis of these assumptions concerning time, inertia and relativity, it is possible to prove that Galilean and Lorentzian invariance are the only two possibilities for transformation groups. Provost's proof sketch proceeds as follows:

Consider a stationary frame R with swiftness ζ, and a second inertial frame R′ undergoing an infinitesimal boost ε in it so that $\zeta' = \zeta - \varepsilon$. I am going to assume the linearity of these transformations; this follows from the homogeneity of spacetime. HB himself outlines two different ways of proving linearity from homogeneity (26-28), one of which is given a general treatment in Lévy-Leblond (1976). Given linearity, the displacements will be transformed as

$$x' = x - \varepsilon x \, f(\zeta) \tag{1}$$

$$f(\zeta') = f(\zeta - \varepsilon) = f(\zeta) - \varepsilon \, df/d\zeta \tag{2}$$

where $f(\zeta)$ is a function of the swiftness ζ with dimension of $[1/\zeta] = [1/v] = L^{-1}T$, so that $x\, f(\zeta)$ is the time for the boost displacement. Now the equality of times principle, Axiom 4, therefore gives:

$$x_3 \, f(\zeta_3) = x_1 \, f(\zeta_1) + x_2 \, f(\zeta_2) \tag{3}$$

Meanwhile

$$x' f(\zeta') = [x - \varepsilon x \, f(\zeta)] \, f(\zeta') \qquad \text{from (1)}$$

$$= [x - \varepsilon x \, f(\zeta)] \, [f(\zeta) - \varepsilon \, df/d\zeta] \qquad \text{from (2)}$$

$$= x \, f(\zeta) - \varepsilon x \, [f^2(\zeta) + df/d\zeta] - o(\varepsilon)^2 \tag{4}$$

and since (3) holds also for $x_1' f(\zeta')$, $x_2' f(\zeta')$, and $x_3' f(\zeta')$, we obtain

$$x_3 \, [f^2(\zeta_3) + df/d\zeta_3)] = x_1 \, [f^2(\zeta_1) + df/d\zeta_1] + x_2 \, [f^2(\zeta_2) + df/d\zeta_2] \tag{5}$$

Compatibility of equations (3) and (5) with $x_1 + x_2 = x_3$ from Axiom 4 requires:

$$f^2(\zeta) + df/d\zeta = \lambda f(\zeta) + \mu \tag{6}$$

where λ and μ are constants. By Axiom 3, the function $f(\zeta)$ must be odd, giving

$$f^2(\zeta) - df/d\zeta = -\lambda f(\zeta) + \mu \quad (7)$$

Adding (6) and (7) gives

$$f^2(\zeta) = \mu \quad (8)$$

$$\Rightarrow df/d\zeta = 0 \quad (9)$$

Subtracting (7) from (6) gives

$$df/d\zeta = \lambda f(\zeta) \quad (10)$$

so that, by (9), λ is identically zero: $\lambda = 0$. (11)

According to (8) we therefore have three cases, corresponding to $\mu = 0$, $\mu > 0$, $\mu < 0$.

In all three cases $t = x f(\zeta)$ for the boost displacement, and since by Axiom 2 this boost displacement x must equal the inertial displacement vt, we have $v = 1/f(\zeta)$. Moreover, from (1) we get

$$x' = x - \varepsilon x f(\zeta) = x - \varepsilon t \quad (12)$$

and from (4) and (6) we obtain

$$t' = x' f(\zeta') = x f(\zeta) - \varepsilon x \mu \quad (13)$$

Equations (12) and (13) are the infinitesimal versions of the group laws for the Galilei group ($\mu = 0$), the Lorentz group ($\mu > 0$), and rotations in spacetime ($\mu < 0$). The lattermost transformation group is ruled out by the causality condition (Axiom 5).[9] We can solve for cases 1 and 2 as follows:

Case 1: $\mu = 0$. $f^2(\zeta) + df/d\zeta = 0$

[9] See Lévy-Leblond (1976, p. 276), Lee and Kalotas 1975, pp. 435-6), Rindler (1977, p. 52) and Lévy-Leblond and Provost (1979, p. 1048).

$\Rightarrow d\zeta = -df/f^2$

$\Rightarrow \zeta = 1/f$ (since f = 0 when $\zeta = 0$)

$\Rightarrow f(\zeta) = 1/\zeta$

Thus $v = 1/f(\zeta) = \zeta$, as Newton had assumed. From (12) and (13) we therefore get the 1-dimensional Galilean transformations:

$$x' = x - vt \qquad (14)$$

$$t' = t \qquad (15)$$

Case 2: $\mu > 0$. $f^2(\zeta) + df/d\zeta = \mu$.

For ease of calculation we let $\mu = 1/k^2$, where k is a positive constant with the dimensions of a velocity. Now

$\Rightarrow d\zeta = df/(1/k^2 - f^2) = k^2 df/(1 - k^2 f^2)$

$\Rightarrow f(\zeta) = 1/k \coth(\zeta/k)$

Thus $v = 1/f(\zeta) = k \tanh(\zeta/k)$.

The constant k is subsequently determined to be the velocity of light in a vacuum, c. If instead of our swiftness ζ we take the dimensionless quantity $\varphi = \zeta/c$, this is the dimensionless group parameter that relativity textbooks define as the *rapidity*, $\varphi = \tanh^{-1}(v/c)$. In terms of the rapidity φ the Lorentz transformation formulas take the simple form:

$$x' = x \cosh \varphi - ct \sinh \varphi$$

$$ct' = ct \cosh \varphi - x \sinh \varphi$$

Harvey Brown, commenting on a related derivation by Jean-Marc Lévy-Leblond (1976) of the Lorentz transformations without any assumptions about light —which he calls the Ignatowski transformations, in honour of their first discoverer[10]— "sounds a warning" about whether the transformations derived "are indeed relativistic in nature" (146, 109). "Unless the magnitude of the invariant speed is established", he writes, "the Ignatowski group can hardly be equated with the Lorentz group". Granted; but as Wolfgang Rindler comments on similar derivations without the light postulate, "the role of a 'second postulate' in relativity is now clear: it merely has to isolate one or the other of these transformation groups. Any second postulate consistent with the RP but not with the GT isolates the LT group", i.e. determines that $\mu = 1/k^2$ —although, as he adds, only a quantitative determination, for instance "at speed 3c/5, there is a time dilation by a factor 5/4", will determine *the* Lorentz group, with $\mu = 1/k^2$ and $k = c$.[11] Also, as Rindler points out, such a quantitative determination can also be obtained in various other ways, from the relativistic mass increase, or from the equivalence of mass and energy ($E = mc^2$), etc. This hardly makes this derivation less relativistic, or indeed less empirical. (Indeed, Provost entitles his paper "A truly relativistic approach [to] the concept of time"!)

The situation is well summarized by Lévy-Leblond in the following passage, and HB's approving quotation of the second sentence (p. 146) and also of the idea of SR as a "super law" (p. 147) seem to signal his agreement:

> All the laws of physics are constrained by special relativity acting as a sort of "super law", and electromagnetic interactions here have no privilege other than a historical and anthropocentric one. Relativity theory, in fact, is but the statement that all laws of physics are invariant under the Poincaré group (inhomogeneous Lorentz group). (1976, p. 271)

Thus, since the Minkowski metric is a straightforward consequence of invariance under the Poincaré group, it has all the empirical content deriving from the Lorentz invariance of all physical laws. Moreover, the preceding Provostian derivation of the

[10] The earliest derivation of the Lorentz transformations without the light postulate were given by W. v. Ignatowski, *Phys. Zeits. 11*, 972 (1910), and by L. A. Pars, *Philos. Mag. 42*, 249 (1921). They were then rediscovered independently by ... Ignatowsky appeared in fictionalized from in the television comedy *Taxi*, in which he was played by Christopher Lloyd.

[11] Rindler 1977, p. 52.

Galilei and Lorentz transformations demonstrates the pedagogical utility of the group theoretic approach, while at the same time showing that the relativity principle does have precise empirical content. It is not necessary to presuppose, as did Newton, that the acceleration is invariant under a boost of inertial frame (relative space). This is the extra assumption pointed out by Barbour, that vitiates Newton's derivation of Corollary 5. Under a Lorentz transformation, acceleration is not invariant. But the non-invariance of accelerations under such a boost is shown by the above analysis to be a consequence of the fact that the velocity is not identical with the rapidity. As Lévy-Leblond and Provost express it in a co-written paper, it shows the need "to replace the Galilean velocity by two separate concepts: 'velocity' v, as expressing the time rate of change of position, and 'rapidity' φ, as the natural additive group parameter." (1979, p. 1045). The Galilean transformations are a kind of degenerate limit of the Lorentzian ones:[12] μ (= $1/c^2$), instead of having a definite positive value, takes the value 0. From this perspective, Newton's (entirely understandable) mistake was not in his assumption that a displacement of a body in absolute space could be effected by an equivalent boost of a body at rest in a relative space —the relativity principle— but in his assumption that such a displacement would thereby be effected in the absolute space in a time equal to that taken by a body moving inertially. Thus on the above Provostian construal, Newton has not made three independent presuppositions —Galilean invariance, the velocity-independence of forces, and the velocity-independence of masses— but only one, namely, the latter one concerning time.

III. Proper Time and the Clock Hypothesis

These latter remarks prompt me to return to the question of the status of SR. In discussing the "miracle of inertia" above, we saw that HB was trenchantly opposed to interpretations of spacetime as an entity which constrains inertial bodies to follow its affine grooves. In this connection he quotes an eloquent passage from Robert DiSalle:

> When we say that a free particle follows, while a particle experiencing a force deviates from, a geodesic of spacetime, we are not explaining the cause of the

[12] Cf. Lévy-Leblond (1976), p. 276: "Our four general hypotheses thus suffice to single out the Lorentz transformations and their degenerate Galilean limit as the only possible inertial transformations."

> difference between two states or explaining 'relative to what' such a difference holds. Instead we are giving the physical definition of a spacetime geodesic. To say that spacetime has the affine structure thus defined is not to postulate some hidden entity to explain the appearances, but rather to say that empirical facts support a system of physical laws that incorporates such a definition. (DiSalle 1995, Brown 2005, p. 25)

In keeping with this, HB insists it "is more natural in theories such as Newtonian mechanics or SR ... to consider the 4-connection as a codification of certain key aspects of the behaviour of particles and fields" (142).

With all of this I am in full agreement. But HB often writes as if the 4-connection or Minkowski metric is purely geometrical, and devoid of physical content.[13] A case in point is his portrayal of *proper time*. This I consider to have been one of the great discoveries of twentieth century physics. I have written elsewhere (2008a) on the metaphysical and physical significance of this bifurcation of the classical time concept into two separate concepts that perform two distinct roles in relativistic physics: the correlating of distant events as prior to, simultaneous with, or after, some given event; and the determining of how fast things age, that is how fast the properties of a given system change, or how fast the states of a given process follow one another. In SR coordinate time performs the first role, and proper time the second. This degeneracy of the classical time concept is, of course, related to the degeneracy of the classical velocity concept discussed above.

Minkowski introduced the concept of proper time in his famous 1908 paper (Lorentz et al. 1923, 73-91). He asked his readers to imagine at any point P (*x, y, z, t*) in spacetime a worldline running through that point, whose magnitude corresponding to the timelike vector *dx, dy, dx, dt* laid off along the line is

$$d\tau = \sqrt{(c^2dt^2 - dx^2 - dy^2 - dz^2)}/c$$

[13] He remarks, for instance: "Mathematically of course the tangent spaces are automatically Minkowskian, but the issue is one of physics, not mathematics." (p. 9)

Then he wrote: “The integral $\tau = \int d\tau$ of this quantity, taken along the worldline from any fixed starting point P_0 to the variable endpoint P, we call the *proper time* of the substantial point at P.” (85)

But compare this with what HB writes:

> If the accelerative forces are small in relation to the internal restorative forces of the clock, then the clock’s proper time will be proportional to the Minkowski distance along its world-line. Consider two events A and B lying on this time-like world-line. The distance along the world-line between these events is given by $\int^{A}_{B} ds$, where $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ in inertial co-ordinates.

Here we see Minkowski’s proper time characterized as “the Minkowski distance” and contrasted with the clock’s proper time. It’s almost as if HB wants to take away the credit from Minkowski and portray proper time as the empirical time, in contrast to the merely geometrical time of the four dimensional representation. Proper time, on HB’s way of looking at it, is the reading we actually get from a clock. It will only agree with the integration of the line element along the object’s path in spacetime if it is true that the ‘restorative effects’ in the clock’s mechanism are not disturbed by its motion along this path. This is the condition referred to as “the clock hypothesis”, the claim that “when a clock is accelerating, the effect of motion on the rate of the clock is no more than that associated with its instantaneous velocity —the acceleration adds nothing” (9):

> [The distance along a worldline between two events] is a sum, in other words, of ‘straight’ infinitesimal elements *ds*: the effect of motion on the clock depends accumulatively only on it[s] instantaneous speed, not its acceleration. This condition is often referred to as the clock hypothesis, and its justification, as we have seen, rests on accelerative forces being small in the appropriate sense. (95)

HB adds that “the term ‘hypothesis’ is arguably a misnomer”, since it tends to mask the fact that whether a clock satisfies the condition is a straightforward dynamical issue: “its justification rests on the contingent dynamical requirement that the external forces accelerating the clock are small in relation to the internal ‘restoring’ forces at work inside the clock.” (115). It is the clock hypothesis that “allows for the identification of the integration of the metric along an arbitrary time-like curve—not just a geodesic—with the proper time. This hypothesis is no less required in general relativity than it is in the special theory.” (9)

But this seems to me a misleading way of describing the state of affairs. If we agree with HB about spacetime or the 4-D connection not being an entity but a “codification of certain key aspects of the behaviour of particles and fields” (142), and that the codification in question here is the “Lorentz invariance of the complete quantum dynamics, known or otherwise, involved in the cohesion of matter” (126), then —provided the empirical evidence supports SR and the clock hypothesis— ideal clocks will conform to Minkowskian geometry. One need not, as HB says J. S. Bell recognized, “know exactly how many distinct forces are at work, nor have access to the detailed dynamics of all of these interactions or the detailed micro-structure of individual rods and clocks” (126). And if ideal clocks locally conform to Minkowskian geometry through the Lorentz invariance of their dynamics, their proper time will be the path integral along an arbitrary timelike line, provided they also conform to the clock hypothesis. It is in this sense that, as both Rindler and HB argue, the “clock hypothesis” may be “regarded as the definition of an ‘ideal’ clock.” (p. 43). It is a separate question whether any real clocks can be found that will measure proper time accurately —that is what I referred to earlier as Harrison’s problem, as opposed to Newton’s. Generally speaking, a real clock will function as an ideal one if its internal driving forces greatly exceed the accelerating force. But if the time kept by such a clock were found to vary with acceleration (over and above the dilation due to the cumulative changes in its instantaneous speed), then this would refute the clock hypothesis. This would indeed require some new dynamics, but it would not show that Minkowski’s proper time was somehow purely geometrical.

“There should be no mystery,” HB concludes, “as to why clocks are waywisers of space-time.” (95) Indeed; and I agree that this is a consequence of the dynamics, rather than due to the action of spacetime or the affine connection on bodies. But it is a result of SR being a kind of “super law” governing the appropriate dynamical reactions, together with the empirical truth of the clock hypothesis, not a consequence of "the operational meaning of the metric [being] ultimately made possible by appeal to quantum theory" (9). We do not need to know the "detailed dynamics of all of these interactions or the detailed micro-structure of individual rods and clocks" in order to know that the dynamical laws are Lorentz invariant. As the Provostian derivation shows, Lorentz invariance can be interpreted as following from the Relativity Principle and certain assumptions about time that are independent of considerations of rods and clocks. And given this Lorentz invariance and the clock hypothesis, we know that clocks will be the waywisers of spacetime.

References

Arthur R. T. W. (1994). "Space and Relativity in Newton and Leibniz", *British Journal for the Philosophy of Science*, **45**, pp. 219-240.

Arthur R. T. W. (1995). "Newton's Fluxions and Equably Flowing Time," *Studies in History and Philosophy of Science*, **28**, no. 2, pp. 323-351.

Arthur R. T. W. (2007). "Beeckman, Descartes and the Force of Motion", *Journal for the History of Philosophy*, **45**, no. 1, 1-28.

Arthur R. T. W. (2008a). "Time Lapse and the Degeneracy of Time: Gödel, Proper Time and Becoming in Relativity Theory," forthcoming in *The Ontology of Spacetime II*, ed. Dennis Dieks, Elsevier.

Arthur R. T. W. (2008b). "On Newton's fluxional proof of the vector addition of motive forces", forthcoming in *Infinitesimals*, ed. William Harper, Wayne C. Myrvold, and Craig Fraser.

Barbour, Julian (1989). *Absolute or Relative Motion? Vol. 1: The Discovery of Dynamics*. Cambridge: Cambridge University Press.

Brown, Harvey (2005). *Physical Relativity: Space-time structure from a dynamical perspective*. Oxford: Clarendon Press.

DiSalle, Robert (1995). "Spacetime theoryas physical geometry", *Erkenntnis* **42**, 317-37.

Einstein, Albert (1954, 1982). *Ideas and Opinions*. New York: Random House.

Ignatowski, W. V. (1910). *Phys. Zeits.* **11**, 972.

Ignatowski, W. V. (1911). *Arch Math. Phys.* **17**, 1, and **18,** 17.

Lange, L. (1886). *Die geschichtliche Entwicklung des Bewegungsbegriff und ihr ausschichtliches Endergebnis*. Leipzig.

Lee, A. R. and Kalotas, T. M. (1975). "Lorentz transformations from the first postulate", *American Journal of Physics 43* (5), 434.

Leibniz, G. W. (2001). *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686*. Ed., sel. & transl. R. T. W. Arthur. New Haven: Yale University Press.

Lévy-Leblond, Jean-Marc (1976). "One more derivation of the Lorentz transformation", *American Journal of Physics 44* (3), pp. 271-276.

Lévy-Leblond, Jean-Marc and Provost, Jean-Pierre (1979). "Additivity, rapidity, relativity", *American Journal of Physics 47* (12), pp. 1045-1049.

Lorentz, H. A., Einstein, A., Minkowski, H. and Weyl, H. (1923), *The Principle of Relativity* (Methuen 1923; reprinted, Dover 1952).

Newton, Isaac (1962). *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his system of the World*. Andrew Motte's 1729 translation, revised and edited by F. Cajori. Berkeley: University of California Press.

Newton, Isaac (1999). *The Principia*: *Mathematical Principles of Natural Philosophy*. Trans. I. Bernard Cohen and Anne Whitman. Berkeley: University of California Press.

Pars, L. A. (1921). *Philosophical Magazine* **42**, 249

Provost, Jean-Pierre (1980). "A truly relativistic approach of the concept of time", pp. 456-458 in *Group Theoretical Methods in Physics*, ed. L. P. Horowitz, Agudah ha-fisiḳalit le-Yiśraʼel. Bristol: A. Hilger.

Rindler, Wolfgang (1977). *Essential Relativity* (2nd ed.). New York: Springer-Verlag.

Robertson, Howard P. and Noonan, Thomas W. (1968). *Relativity and Cosmology*. (Philadelphia-London-Toronto: W. B. Saunders).

Sobel, Dava (1995). *Longitude: The true Story of a Lone Genius Who Solved the Greatest Scientific Problem of his Time*. (London: Fourth Estate).