Zeno of Elea

In his dialogue *Parmenides* Plato informs us that Zeno was a companion of Parmenides, whose arguments he defended against his detractors. A young Socrates accuses Zeno of being deceptive, misleading people into thinking he was arguing for something different from Parmenides, when in fact by arguing against the existence of several things or against motion he is effectively arguing for Parmenides' conclusion that reality is indivisible, unchanging, and one. This signals the novelty of the form of argument Zeno has introduced into philosophy for the first time. This is what is now called the *reductio ad absurdum*, where one argues for a view by showing that the opposite entails a contradiction: on the hypothesis that there are many things, for instance, Zeno argues that “they are both small and large—so small as not to have a magnitude, and so large as to be infinite.”

Zeno is said to have written a book containing about forty such arguments when he was young, that is, in about 475 BCE. Aristotle gives paraphrases of four of the arguments against the reality of motion, and Simplicius gives three substantial fragments (B1-B3) of two of the arguments against plurality in Zeno’s own words. In the first argument against plurality he argues that if many things exist, they must be both finitely many and yet infinitely many; Simplicius quotes him as follows:

If many things exist, it is necessary for them to be as many as they are, and neither more nor fewer. But if they are as many as they are, they will be finite. If many things exist, the things that exist are infinite. For there are always others between the things that exist, and again others between them. And in this way the things that exist are infinite. (B 3)

In his second argument against plurality, Zeno argues that if many things exist, they must have no magnitude, and yet have infinite magnitude, as mentioned above. He begins by proving that “nothing has magnitude because each of the many is self-identical and one.” A conjectural reconstruction of this lost text is as follows:

<If each thing that exists is self-identical and one, it will not have a magnitude. For if it has magnitude, it will have thickness, and will therefore have parts. But nothing with parts can be one. Hence if many things exist, none of them will have magnitude.>

The argument continues with an argument that what has no magnitude cannot, however, even exist. For, quotes Simplicius,

if it were added to anything else, it would not make it larger. For if it is of no magnitude but is added to another, it would not increase the other’s magnitude. Thus what is added will be nothing. And if when it is subtracted the other thing is made no smaller, nor increased when it is added, it is clear that what is added or subtracted is nothing. (B 2)

In the other fragment quoted by Simplicius, the argument continues:

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1 The *reductio* form first appears in Pythagorean mathematics in the famous proof that the number whose square is 2 cannot be rational. Socrates gives it a more *ad hominem* interpretation, with his famous *elenchus*, where the opponent is reduced to admitting that he does not know what he thought he did.
But if many things exist, each must have some magnitude and thickness, and one of its parts must extend beyond another. And the same argument holds for the projecting part. For that too will have magnitude, and part of it too will project. Now it is all one to say this once and to say it forever. For it will have no last part of such a sort that there is no longer one part in front of another.

In this way, if there are many things, they must be both small and large—so small as not to have a magnitude, and so large as to be infinite. (B 1)

Another purported fragment is given by the unreliable Diogenes Laertius as a fifth Zenonian argument against motion: (B4): *What is moving is moving neither in the place in which it is nor in the place in which it is not.* A sixth argument against motion is reported by both Aristotle and Simplicius, according to which whatever exists must exist in some place, with the result that places themselves do not exist. Both authors also report the paradox of the millet seed, according to which the sound made by a bushel of seed must have the same ratio to an individual seed (or ten-thousandth part of one) as their weights; yet an individual seed (or ten-thousandth part of one) makes no noise.

Aristotle’s paraphrases of the first three of Zeno’s “four arguments about motion which have proved troublesome for people to solve” are as follows (*Physics*, 239b10-32):

The first maintains that nothing moves because what is travelling must first reach the half-way point before it reaches the end. We have discussed this earlier. [Zeno’s argument assumes that it is impossible to traverse what is infinite, or to touch infinitely many things one by one in a finite time. But this assumption is false. For both lengths and times—and indeed all continua—are said to be infinite in two ways: either by division or in respect of their extremities. So although it is impossible to touch things infinite in quantity in a finite time, it is possible so to touch things infinite by division. For time itself is infinite in this way. Hence it follows that what is infinite is traversed in an infinite and not in a finite time, and that the infinitely many things are touched at infinitely many, and not finitely many, nows. (233a21-31)]

The second is the so-called Achilles. This maintains that the slowest runner will never be caught by the fastest. For the pursuer must first reach the point where the pursued set out, so that the pursued must always be ahead. This is the same argument as the dichotomy, but it differs in that the additional magnitudes are not divided in half. Now it follows from the argument that the slower is not caught, but this depends on the same point as the dichotomy: in both cases it follows that it is impossible to reach the end if the magnitude is divided in a certain way (but here there is the additional feature that not even the fastest runner in the story can reach his goal when he pursues the slowest); hence the solution must also be the same. It is false to claim that the one ahead is not caught; it is not caught while it is ahead, but nonetheless it is still caught (provided one grants that they can traverse a finite distance).

So much for two of the arguments. The third is the one we have just stated, to the effect that a moving arrow stands still. [Zeno argues fallaciously. For if, he says, everything is always at rest when it is in a space equal to itself, and if what is travelling is always in such a space at any instant, then the travelling arrow is motionless. That is false; for time is not composed of indivisible nows—nor is any other magnitude. (239b5-239b8)] Here the conclusion depends on the assumption that time is composed of nows; if this assumption is not granted, the argument fails.
The fourth is the argument about bodies in a stadium moving from opposite directions, an equal number past an equal number; one group starts from the end of the stadium, the other from the middle, and they move at the same speed. The result, according to Zeno, is that half the time is equal to its double. The fallacy consists in supposing that bodies of equal sizes moving at the same speed, the one past a moving body and the other past a body at rest, travel for an equal time. This is false. For example, let AA be the equal stationary bodies; let BB be those beginning from the middle, equal to them in number and size; and let CC be those beginning from the end, equal to them in number and size and moving with the same speed as the BB. Now it follows that the first B and the first C, as the two rows move past each other, will reach the end of each other’s rows at the same time. And from this it follows that although the first C has passed all the Bs, the first B has passed half the number of As. Hence the time is half, because each of the two is alongside each for an equal time. And it also follows that the first B has travelled past all the Cs; for the first C and the first B will be at opposite ends at the same time (since, as he says, the first C spends the same amount of time alongside each of the Bs as it does alongside each of the As), because both are alongside the As for an equal time. That is the argument, and it depends upon the fallacy we have mentioned.

Finally two arguments against divisibility ascribed by Porphyry to Parmenides, are probably—as Alexander of Aphrodisias and Simplicius judged—actually due to Zeno. The first exploits an argument by dichotomy to show that if what exists is divisible, it will be divisible into halves “forever”, so that it will either finally consist in an infinity of minimal magnitudes, or “will disappear and be dissolvable into nothing”. Thus we have either an infinite magnitude, or an endless supply of nothings from which the divided thing could never be constituted. This echoes the “either so small as not to have a magnitude, or so great as to be infinite” quoted by Simplicius. The second argument supposes that what exists is divisible everywhere alike:

Then let it have been divided everywhere. It is clear again that nothing will remain but that it will disappear; and if it is constituted at all, it will again be constituted from nothing. For if anything remains, it will not yet have been divided everywhere. thus form these considerations too it is evident, he says, that what exists will be indivisible and partless and one.

Prepared by RTWA, January 2007